INFORMATION ABOUT SECOND KYIV–WARWICK STOCHASTIC ANALYSIS SEMINAR

The seminar took place online at December 10, 2024. It was devoted to modern problems of stochastic analysis and stochastic flows theory. The organizers of the seminar were Prof. Andrey A. Dorogovtsev, Prof. Roger Tribe and Prof. Oleg Zaboronski. Here we publish the programme and abstracts of the talks.

(1) **Andrey Dorogovtsev**, *Folding phenomena in random knot evolution.*
In the talk the evolution of the smooth curve in the stochastic flow. We discuss posibility not only change the topological type, but also folding of the curve and speed of this process.
Reference. Dorogovtsev, Andrey A. (2023) *On a Stationary Random Knot*, Journal of Stochasic Analysis: Vol. 4, No. 3, Article 4.

(2) **David Elworthy**, *Stochastic differential equations and higher order derivatives of heat semigroups.*
For the heat semigroup on a compact, or more general, Riemannian manifold M there is the celebrated result of Bakry and Emery giving the exponential growth of the first spatial derivative of the heat semigroup in terms of the lower bound of the Ricci curvature of M, and showing this characterises that lower bound. An easy way to see that the exponential bound holds is to represent the semigroup using a stochastic flow of a suitable SDE, differentiate under the expectation, and use the technique of removal of "redundant noise". Different SDE's give different answers, or give difficulties in estimation. Gradient SDE's give the optimal result. In theory this technique works for higher derivatives. For spheres, considered as symmetric spaces, one gets complete looking answers for arbitrary high covariant derivatives, but the exponential behaviour depends on the directions of differentiation. The same approach should work for compact symmetric spaces, though the results may depend on the representation of M as a symmetric space.

For general M, to see how the C^r norm of the solution of the heat equation depends on the C^r norm of its initial condition it seems sensible to consider jets of functions, essentially truncated Taylor series, instead of individual covariant derivatives. In this talk we will describe the positive results and techniques for spheres and symmetric spaces with open questions, and show how problems arise in the general case. References to other work as well as the results for spheres can be found in:
Reference. Elworthy, K. D. (2022) Higher order deriviatives of heat semigroups on spheres and Riemannian symmetric space. In: Ugolini, Stefania and Fuhrman, Marco and Mastrogiacomo, Elisa and Morando, Paola and Rüdiger, Barbara, (eds.) Geometry and Invariance in Stochastic Dynamics: Verona, Italy, March 25-29, 2019. Springer Proceedings in Mathematics and Statistics, 378. Cham: Springer Nature, pp. 113-136. ISBN 9783030874315

(3) **Karen Habermann**, *Long-time existence of Brownian motion on configurations of two landmarks.*
In computational anatomy and, more generally, shape analysis, the Large Deformation Diffeomorphic Metric Mapping framework models shape variations as diffeomorphic deformations. An important shape space within this framework is the space consisting of shapes characterised by $n \geq 2$ distinct landmark points in $\mathbb{R}^d$. In diffeomorphic landmark matching, two landmark configurations are

compared by solving an optimisation problem which minimises a suitable energy functional associated with flows of compactly supported diffeomorphisms transforming one landmark configuration into the other one. The landmark manifold Q of n distinct landmark points in $\mathbb{R}^d$ can be endowed with a Riemannian metric g such that the above optimisation problem is equivalent to the geodesic boundary value problem for g on Q. Despite its importance for modelling stochastic shape evolutions, no general result concerning long-time existence of Brownian motion on the Riemannian manifold (Q, g) is known. I will present joint work with Philipp Harms and Stefan Sommer on first progress in this direction which provides a full characterisation of long-time existence of Brownian motion for configurations of exactly two landmarks, governed by a radial kernel.

(4) **Vitalii Konarovskyi**, *A quantitative central limit theorem for the simple symmetric exclusion process.*
We will discuss a quantitative central limit theorem for the simple symmetric exclusion process on a multidimensional discrete torus. Our argument is based on a comparison of the generators of the density fluctuation field of the symmetric exclusion process and the generalized Ornstein-Uhlenbeck process, as well as on an infinite-dimensional Berry-Essen bound for the initial particle fluctuations. The obtained rate of convergence is optimal. It is a joint work with Benjamin Gess (TU Berlin).

(5) **Georgii Ryabov**, *Stochastic flows of kernels.*
In the talk the notion of stochastic flow of kernels will be discussed. Examples of SDE's whose solutions can be described by stochastic flows of kernels will be given. A general theorem about characterization of distributions of stochastic flows of kernels will be presented.

(6) **Daniel Valesin**, *Catalan percolation.*
In Catalan percolation, all nearest-neighbor edges $\{i, i+1\}$ along the integer line are initially occupied, and all other edges are open independently with probability p. Open edges $\{i, j\}$ are occupied if some pair of edges $\{i, k\}$ and $\{k, j\}$, with $i < k, j$, become occupied. This model was introduced by Gravner and Kolesnik, in the context of polluted graph bootstrap percolation. We prove that the critical p_c is strictly between that of oriented site percolation on $\mathbb{Z}^2$ and the Catalan growth rate $1/4$. Our main result shows that an enhanced oriented percolation model, with non-decaying infinite-range dependency, has a strictly smaller critical parameter than the classical model. This is reminiscent of the work of Duminil-Copin, Hilário, Kozma and Sidoravicius on brochette percolation. Our proof differs, however, in that we do not use Aizenman–Grimmett enhancements or differential inequalities. Two key ingredients are the work of Hilário, Sá, Sanchis and Teixeira on stretched lattices, and the Russo–Seymour–Welsh result for oriented percolation by Duminil-Copin, Tassion and Teixeira. This is a joint work with Eleanor Archer, Ivailo Hartarsky, Brett Kolesnik, Sam Olesker-Taylor, and Bruno Schapira.

(7) **Mykola Vovchanskii**, *On approximations of finite-dimensional point densities for Arratia flows with drift.*
Representations of finite-dimensional points densities for Arratia flows with drift in the terms of Gaussian densities, special stochastic exponentials involving Brownian bridges and Karlin-McGregor determinants are given. As applications, approximations of such densities by discretization of the starting interval and a limit theorem for the convergence of drift coefficients are obtained.

(8) **Jon Warren**, *The stability of Brownian maxima and extending the noise of splitting.*
Suppose B is a one-dimensional Brownian motion and let T be some (closed) subset of $\mathbb{R}_+$ thought of as a set of times. Perturb B by replacing its increments

on T^c by an independent copy whilst keeping its increments on T unchanged. When does a local maximum of B which falls in T survive the perturbation process i.e. remain a local maximum? It depends on the structure of T, and in particular the asymptotic behaviour of its local density. As an application we can describe fairly explicitly how a specific example in Tsirelson's general theory of non-classical noises – associated with a stochastic flow constructed from Tanaka's SDE – can be extended.