

DMITRII SILVESTROV AND ANATOLIY MALYARENKO

THE ANALYTICAL FINANCE PACKAGE

We describe the Analytical Finance Package, a set of Java applets which is developing at the Mälardalen University.

1. INTRODUCTION

Analytical Finance is the research area that includes financial mathematics, financial engineering, and financial and risk management software. The Analytical Finance group was created by the Department of Mathematics and Physics at the Mälardalen University in 1999. Research studies of the group are concentrated in the above mentioned domains and in some related research areas such as actuarial mathematics, optimisation, applied statistics and stochastic processes, computational game theory, simulation, scientific computing, informatics.

The development of the pilot financial and risk management software projects is one of the main research area of the Analytical Finance Group. In this paper, we describe a project “Analytical Finance Package”.

The Analytical Finance Package is a library of applets in the area of Analytical Finance. The project was initiated in 2003, and is developing on a permanent base. The library is free software and consists mainly of the applets written by the students as their seminar reports, bachelor and master theses. You can find the applets and their documentation on the Web page [1]. In order to make your browser adapted for running applets, you may need to download the Java Runtime Environment for your platform from <http://java.sun.com>.

The applets are divided into three groups: **A** Simulation of pricing processes; **B** Estimation of pricing processes; **C** Evaluation of financial contracts.

In what follows, we will describe the library in more details.

Invited lecture.

2000 *Mathematics Subject Classifications*. 62P05.

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2. SIMULATION OF PRICING PROCESSES

The students of the program “Analytical finance” who studied Java in 2004, have written applets concerning computer simulation of stochastic processes. The resulting applets are:

1. Applet A01: Autoregressive model, by Krasimira Kirova and Ying Ni. This applet simulates the autoregressive model of order 4, where the logarithmic returns h_n are $h_n = a_0 + a_1h_{n-1} + a_2h_{n-2} + a_3h_{n-3} + a_4h_{n-4} + \sigma\varepsilon_n$, with some constants a_0, a_1, a_2, a_3, a_4 , and σ , where ε_n is the sequence of independent standard normal random variables. Ying Ni is currently a PhD student of the Department of Mathematics and Physics.
2. Applet A02: Jump-diffusion model, by Robin Lundgren. This applet simulates the Merton jump-diffusion model:

$$S(t) = S(0) \exp \left\{ (\mu - \sigma^2/2 - \lambda\nu)t + \sigma W(t) + \sum_{j=1}^{N(t)} Y_j \right\},$$

where the Wiener process $W(t)$, the Poisson process with intensity λ , and the identically distributed random variables Y_j are mutually independent. Later on, this project was extended to the master thesis. Robin Lundgren is currently a PhD student of the Department of Mathematics and Physics.

3. Applet A03: Standard user interface for simulation applets, by Xin Mai and Weiss Amani. The authors created a user interface for simulation applets. It contains simulation of standard Cox–Ross–Rubinstein model and pricing index controlled by Markov chain.
4. Applet A04: GARCH model, by Sona Gevorgyan, Enrike Barrientos and Nahir Hanna. This applet simulates the Generalised Autoregressive Conditional Heteroscedasticity model, where past observations of the variance and variance forecast are used to predict future variances: $h_n = \sigma_n\varepsilon_n$ with $\sigma_n^2 = a_0 + \sum_{j=1}^p a_j h_{n-j}^2 + \sum_{j=1}^q b_j \sigma_{n-j}^2$.
5. Applet A05: ARMA model, by Gao Jongjie and Rafael Cortes. This applet simulates the ARMA model $h_n = a_0 + \sum_{j=1}^p a_j h_{n-j} + \sum_{j=1}^q b_j \varepsilon_{n-j} + \sigma\varepsilon_n$.
6. Applet A06: GPI model, by Herve Fandom Tchomgouo. This applet simulates trajectories of the pricing process controlled by global price index,

$$h_n = a_0 + \sum_{j=1}^p a_j h_{n-j} + \sum_{j=1}^q b_j \tilde{h}_{n-j} + \sigma_n \varepsilon_n,$$

where \tilde{h}_n is the pricing index generated by another sequence of independent standard normal random variables $\tilde{\varepsilon}_n$, independent from ε_n :

$$\tilde{h}_n = a_0 + \sum_{j=1}^q \tilde{a}_j \tilde{h}_{n-j} + c_n \tilde{\varepsilon}_n.$$

7. Applet A07: Stochastic volatility model, by Daniela Andersson and Zheng Wang. This applet simulates the following model:

$$h_n = \sigma_n \varepsilon_n,$$

where

$$\sigma_n = \exp[(a_0 + a_1 \Delta_{n-1} + \dots + a_p \Delta_{n-p} + c \tilde{\varepsilon}_n)]/2.$$

8. Applet A08: Cox–Ross–Rubinstein model, by Mazyar Rostami. This now classical model is:

$$h_n = \varepsilon_n \ln \lambda,$$

where λ and ε_n is the sequence of independent and identically distributed Bernoullian random variables with

$$P\{\varepsilon_1 = 1\} = p, \quad P\{\varepsilon_1 = -1\} = 1 - p, \quad 0 < p < 1.$$

9. Applet A09: automaton model simulator, by Robert Byström. The model under simulation is

$$h_n = \mu_i + \sigma_i \varepsilon_n, \quad \text{if } I_n = i,$$

where $I_n = 1$, if $h_{n-1} \Delta$, $I_n = 0$, if $|h_{n-1}| \leq \Delta$, and $I_n = -1$, if $h_{n-1} - \Delta$.

10. Applet A10: moving average model, by Alexander Svahn, David Hefner, Jakob Wernroth and Jonas Gustavsson. The model is

$$h_n = b_0 + b_1 \varepsilon_{n-1} + \dots + b_q \varepsilon_{n-q} + \sigma \varepsilon_n.$$

3. SOLUTIONS TO SOME ELEMENTARY EXERCISES IN MATHEMATICAL FINANCE

The students of the program “Analytical finance” who studied Java in 2005, have written applets that solved some calculational exercises from [2]. The resulting applets are:

1. Applet C01: American options with dividends, by Ling Wang, Bing Wang and Yanjun Wang. Solves exercises 7.1–7.3.

2. Applet C03: pricing European options by binomial model method, by Tina Vedenpää and Cecilia Flink. Solves exercises 5.1–5.2.
3. Applet C04: pricing European call options on a forward contract, by Mahsima Ranjbar, Malin Andersson and Cecilia Isaksson. Solves exercises 7.7–7.9.
4. Applet C05: binomial pricing of European put options with replication, by Armand Fotsing, Hamadou Hamaoude, David Wellton, Basil Wakid Hassan and Ren Minyi. Solves exercises 5.8–5.9.
5. Applet C06: replicating the stock in the binomial pricing model, by Fred Takoeta. Solves exercise 5.6.
6. Applet C07: pricing American put options with replication, by Antti Laine, Amir Kheirollah and Toma Boyacioglu. Solves exercises 7.4–7.6.
7. Applet C09: bond price calculator, by Aminur Roshid, Isaac Acheampong, Peter Agyemang-Mintah and Yue Song. Solves exercises 1.6–1.10.

Later on, these applets were included in the Internet-based lecture notes “Introduction to mathematical finance” by the second author.

4. SOLUTIONS TO SOME PROBLEMS FROM INSTITUTE OF ACTUARIES EXAMINATION PAPER

The students of the program “Analytical finance” who studied Java in 2006, have written applets that solved some problems from Institute of Actuaries examination papers [3]. The resulting applets are:

1. Applet C11: Eurobonds problem, by Kamila Giedrojć, Ewa Tropak and Romans Obrezkovs. Examination 7 September 2005, subject CT1 — Financial Mathematics Core Technical, exercise 6.
2. Applet C12: investor’s problem, by Elizabeta Tudzarovska, Izabela Matusiak, Kwok-wai Choy, Sophia Abdi Hassan and Khadija Khapasi. Examination 6 April 2005, subject CT1 — Financial Mathematics Core Technical, exercise 7.
3. Applet C13: bond reselling problem, by Malgorzata Andros, Matti Simperi and Piotr Liszewski. Examination 7 September 2005, subject CT1 — Financial Mathematics Core Technical, exercise 10.

4. Applet C14: loan repayment applet, by Osei Benjamin Kwesi Amoako, George Manteaw Anobah, Aleksandra Sroka and Jacek Zniszczoł. Examination 6 April 2005, subject CT1 — Financial Mathematics Core Technical, exercise 11.
 5. Applet C15: insurance problem, by Beata Lubecka, Peter Malosha Mayunga, Maziar Saei Aghmiouni and Marek Geringer De Oedenberg. Examination 7 September 2005, subject CT1 — Financial Mathematics Core Technical, exercise 11.
5. SOLUTIONS TO PROBLEMS ABOUT CALCULATION OPTION PRICES BY FINITE DIFFERENCE METHODS

The students of the program “Analytical finance” who studied Java in 2007, have written applets that solved some exercises concerning calculation option prices by finite difference methods from [4]. The resulting applets are:

1. Applet C27: European Option Calculator, by Jie Zhang, Michael Wennermo, Tatiana Ozhigova, Konrad Szczyrkowski, Michail Musatov. Solves exercise
2. Applet C28: Java Applet for pricing European options using implicit finite-difference method, by Boyko Vasilev, Mbecho Techago, Hanney Al-Qaisi, Viktor Taku-Mbi, Vitaliy Drozdenko.
3. Applet C29: Java Applet for pricing American options using projected explicit finite-difference method, by Zhi Xu, Tian Tian, Wang Zheng, Michail Kalavrezos, Mehmet Yasin Hürata.
4. Applet C30: Java Applet for the pricing of American options using the implicit finite-difference method, by Dong Liang, Leonel Taku Ayuk, Takura Muusha, and Coline Sume Emadione.

6. BACHELOR THESES

Several students of the Analytical Finance program have chosen to write their bachelor theses using Java. Their applets are:

1. Applet C16: Pricing put options using explicit finite difference method in Java graphical user interface, by Yue Song. The algorithm from [5, pp. 92–95] was translated to Java, the graphical user interface was written, the numerical experiments were performed.
2. Applet C18: Pricing futures using the two-period binomial model in Java, by Minyi Ren and Wakid Hassan Basil. This study develops an applet for binomial futures pricing.

3. Applet C24: Monte Carlo Simulation of Bond Prices in the Ho and Lee Model, by Sophia Abdi Hassan. The algorithm from [6] was realised in Java, the graphical user interface was written, the numerical experiments were performed.
4. Applet C26: Java Applet For The Closed Form Valuation Of American Option Using Bjerksund and Stensland Model, by Mbecho Techago Emmanuel. The algorithm from [7] was realised to Java, the graphical user interface was written, the numerical experiments were performed.

We would like to describe Applet C26 in more details. Bjerksund and Stensland [7] obtain an accurate and computer efficient lower approximation to the American option value by imposing a feasible but non-optimal exercise strategy. In particular, they assume a flat early exercise boundary. In [8], they divided time to maturity into two periods, each with a flat early exercise boundary, and obtained even more accurate lower approximation. They also suggested a reasonable approximation of the true option value, twice the option value calculated by the two-step boundary method minus the option value calculated by the flat boundary method.

Mbecho Techago Emmanuel realised all the three above described methods plus the binomial tree method in his applet and has written a graphical user interface. A typical result of calculation is shown in Fig. 1.

7. MASTER THESES

Several students of the Analytical Finance program have chosen to write their master theses using Java. Their applets are:

1. Applet A11: Java Applet for the Pricing of Exotic Options by Monte-Carlo Simulations in a Lèvy market with Stochastic Volatility, by Isaac Acheampong. The algorithm from [9] was realised to Java, the graphical user interface was written, the numerical experiments were performed.
2. Applet C08: The Hull-White model, by Aminur Roshid. The algorithm from [6] was realised in Java, the graphical user interface was written, the numerical experiments were performed.
3. Applet C10: Simulation of the short interest rate in the Vasicek model, by Natalia Spas'ka and Olexander Sheychenko. The algorithm from [6] was realised in Java, the graphical user interface was written, the numerical experiments were performed. The authors are currently working in a big insurance company in Sidney.

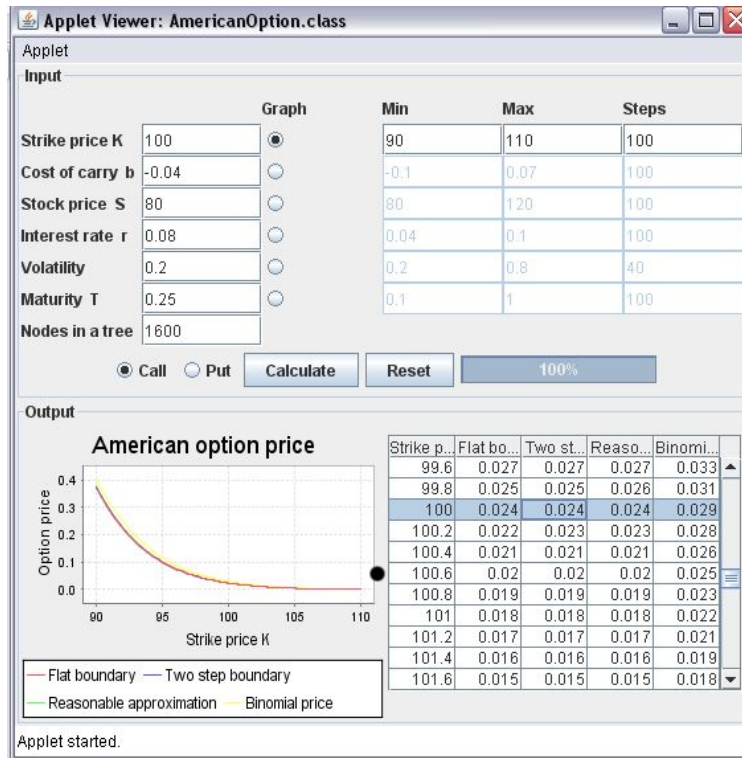


Figure 1: Typical results of calculations in Applet C26

4. Applet C17: A Java program for pricing options using the trinomial tree, by Youmbi Etien Kalame. An applet for options pricing was written.
5. Applet C19: Pricing convertible bonds with Monte Carlo simulations, by Cecilia Isaksson. The algorithm from [10] was realised in Java, the graphical user interface was written, the numerical experiments were performed. The author is currently working in a bank in Liechtenstein.
This work was continued in Applet C23 by Kateryna and Vladimir Mishchenko. The results of their work are published in this volume.
6. Applet C20: A Java applet for credit risk estimation with Wishart multivariate stochastic volatility, by Amoako Osei Benjamin Kwesi. The algorithm from [11] was realised in Java, the graphical user interface was written, the numerical experiments were performed.
7. Applet C21: A Java applet for pricing convertible bonds with credit risk, by Charles Etang Ntui. The algorithm from [12] was realised in Java, the graphical user interface was written, the numerical experiments were performed.

8. Applet C22: A Java applet for simulation of economy with borrowers under costly defaults, by Basil Wakid Hassan. The algorithm from [13] was realised in Java, the graphical user interface was written, the numerical experiments were performed.
9. Applet C25: The Java applet for pricing put options by the implicit finite difference method, by Wang Janjun. Several numerical finite difference methods were realised in this applet.

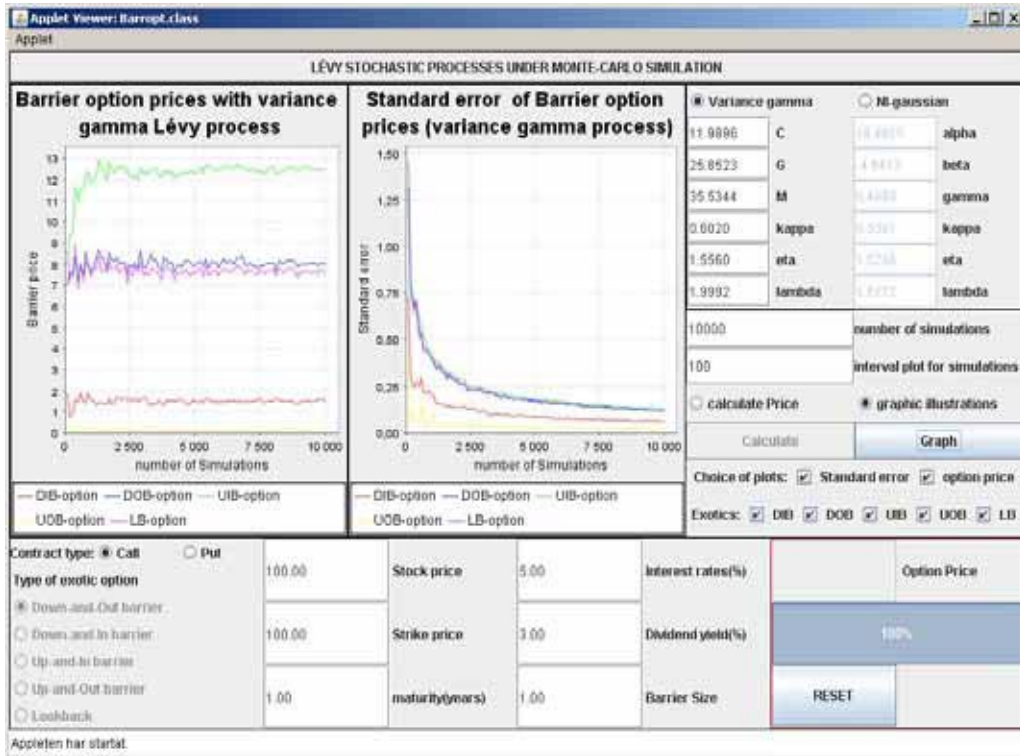


Figure 2: Typical results of calculations in Applet A11

We would like to describe Applet A11 in more details. Schoutens and Symens [9] price barrier, lookback and cliquet options by Monte-Carlo simulation in a stock price model based on Lévy processes with stochastic volatility. The sampling of paths is based on a compound Poisson approximation of the Lévy process involved.

Isaac Acheampong realised this complicated model in Java. A typical result of calculations is shown in Fig. 2.

8. CONCLUSIONS

In this paper, we have described the Analytical Finance Package, which is creating by the students of the Analytical Finance program at the the

Mälardalen University under the authors' supervision. In future, we plan to further develop this useful programming tool and include several interesting stochastic models of financial instruments.

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DEPARTMENT OF MATHEMATICS, MÄLARDALEN UNIVERSITY, BOX 883,
SE-72123, VÄSTERÅS, SWEDEN

E-mail: DMITRII.SILVESTROV@MDH.SE, ANATOLIY.MALYARENKO@MDH.SE