Theory of Stochastic Processes
Vol. 23 (39), no. 2, 2018, pp. 1–6

N. ALEMOHAMMAD

VALUE AT RISK FORECASTING OF GOLD PRICE: A COMPARISON BETWEEN THE GARCH AND LST-GARCH MODELS

Value at risk is one of the most important measure in finance. This paper evaluates the value at risk forecasting performance of the GARCH and logistic smooth transition GARCH (LST-GARCH) models for the gold markets. The LST-GARCH model is capable to react differently to positive and negative shocks in financial time series. The results show that the LST-GARCH structure provides the more adequate value at risk forecasts relative to the GARCH model.

1. Introduction

Financial time series generally relieve some particular specifications that must be considered when the objective is to forecast the future risks. Some of these characteristics are as:

- They are usually non-stationary.
- There is almost no significant correlation between financial time series.
- The squared of the observations are strongly correlated.
- In a financial time series, conditional variance isn't constant over time.
- They almost depict leverage effect property i.e. the conditional variance of series reacts differently to positive and negative shocks with the same absolute values, [17].

In the past few decades, there has been a growing interest in volatility modeling of financial time series. The ARCH and GARCH models, introduced by Engle [7] and Bollerslev [4], are the most famous structures to model volatility.

One limitation of the GARCH model is a symmetry reaction to the sign of past shocks. Financial markets become higher volatile in response to negative shocks relative to positive one. A generalization of the GARCH model is a case that conditional volatility to be a function of size and sign of the past observation. Study of the asymmetric GARCH structure started by Engle [8] and continued as the Exponential GARCH (EGARCH) model by Nelson [14], GJR-GARCH model by Glosten, et al.[9] and Threshold GARCH (TGARCH) model by Zakoian [18]. The other asymmetric structures are smooth transition models introduced by Lubrano [12], Ardia [2], Medeiros and Veiga [13] and Haas et al. [11].

Gold has played an important role on the world economy and the relation between it and financial components is corroborated, [15]. Recently there has been great consideration to global gold market. To relieve the risk in gold price oscillations, it is necessary to handle some nonlinear models to capture its dynamics. Truck [16] and Liang showed that TGARCH model provide the best volatility forecasting. Value at Risk has become one of the most important risk measurement technique in finance. It measures the potential loss of a financial position during a given time period for a given confidence interval [17]. Cheng et al. [5] used the power GARCH model to obtain out of sample VaR estimate.

2000 *Mathematics Subject Classification.* 62M10; 91B84.
Key words and phrases. Forecasting, Smooth transition GARCH, Leverage effect, Value at Risk.

In this paper, we apply the GARCH and LST-GARCH models to forecast the value at risk of gold market. The LST-GARCH model obviates the absence of asymmetric property in the GARCH model by considering a smooth weight in its structure [12].

The parameters of the models are estimated by applying MCMC methods through Gibbs and griddy Gibbs sampling. We illustrate the out-of-sample forecasting performance of one-day-ahead value at risk of the proposed models for the daily gold returns from 2004 to 2018.

The plan of this paper is as follows: the GARCH and LST-GARCH models are presented in section 2. Section 3 is devoted to a detailed explanation of the VaR. The results of empirical study are discussed in Section 4. Section 5 concludes.

2. Nonlinear models

In this section two nonlinear models GARCH and LST-GARCH are studied.

GARCH

The Generalized Autoregressive Conditional heteroscedasticity model of order p and q for time series $\{y_t\}$, GARCH(p,q), is introduced as

$$y_t = z_t\sqrt{h_t}$$

where $\{z_t\}$ is a white noise process with zero mean and variance σ^2 and h_t is the conditional variance of $\{y_t\}$ that is defined as

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}.$$

Sufficient conditions to warrent strictly positive h_t are $\omega > 0$, $0 \leq \alpha_i < 1$, $0 \leq \beta_j < 1$, $(i = 1, \cdots, p; j = 1, \cdots, q)$.

LST-GARCH

The logistic smooth transition GARCH model (LST-GARCH) for the time series $\{y_t\}$ is introduced as:

$$y_t = z_t\sqrt{h_t},$$

$$h_t = \omega + \alpha_1 y_{t-1}^2(1 - w_{t-1}) + \alpha_2 y_{t-1}^2 w_{t-1} + \beta h_{t-1}, \tag{1}$$

where the weights (w_{t-1}) is the logistic function of the past observation as

$$w_{t-1} = \frac{1}{1 + \exp(-\gamma y_{t-1})} \qquad \gamma > 0, \tag{2}$$

which is monotonically increasing with respect to previous observation and is bounded, $0 < w_{t-1} < 1$. The parameter $\gamma > 0$ is called the slope parameter. The weight function w_{t-1} goes to one when $y_{t-1} \to +\infty$ and so w_{t-1} tends to one. Also it goes to zero when $y_{t-1} \to -\infty$. Therefore the effect of large negative shocks are mainly described by α_1 and of positive shocks by α_2. For modeling financial leverage effect it is assumed that $\alpha_1 > \alpha_2$. This enables one to provide a flexible model for describing such different transitions. Figure 1 plots logistic weight functions for the returns of gold data. In this model conditional variance is under the influence of size and sign of shocks. Indeed in each regime the coefficient of y_{t-1}^2 is time dependent that causes the volatility structure being under the influence size and sign of the observations and it makes distinct from GARCH model. When γ_j grows to zero, $w_{j,t-1}$ goes to 1/2 and the LST-GARCH model tends to the GARCH model.

Sufficient conditions to guarantee strictly positive conditional variance (1) are that ω to be positive and α_1, α_2, and b_j being nonnegative.

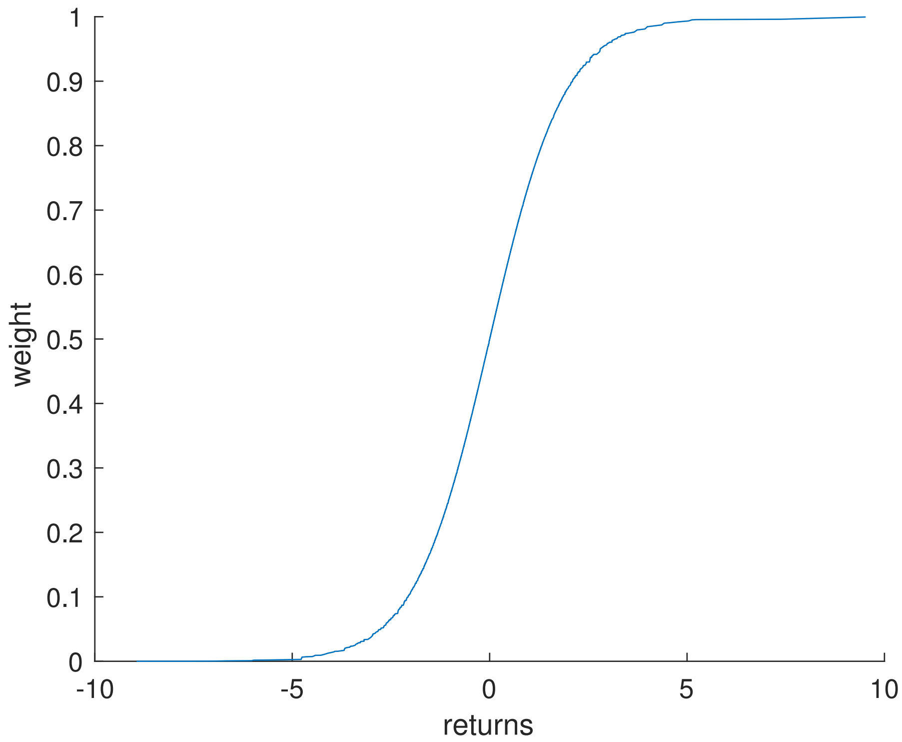

FIGURE 1. Logistic weight function

3. VALUE AT RISK

The one-day-ahead value at risk level $\alpha \in (0,1)$, VaR(α) is obtained by calculating the $(1-\alpha)$th percentile of the one-day-ahead predictive distribution [1]. To test the VaR at level α, we define the sequence $\{V_t(\alpha)\}$ by

$$V_t(\alpha) = \begin{cases} I\{y_{t+1} < VaR(\alpha)\} & \text{if } \alpha > 0.5 \\ I\{y_{t+1} > VaR(\alpha)\} & \text{if } \alpha \leq 0.5. \end{cases}$$

The out-of-sample VaR at level α has good performance if the sequence $\{V_t(\alpha)\}$ are independent and obey the following distribution

$$V_t(\alpha) \sim \begin{cases} Bernoulli(1-\alpha) & \text{if } \alpha > 0.5 \\ Bernoulli(\alpha) & \text{if } \alpha \leq 0.5, \end{cases}$$

The three likelihood ratio statistics for unconditional coverage (LR_{uc}), independence (LR_{ind}) and conditional coverage (LR_{cc}) tests are as follows [6]:
1. LR statistic for the test of unconditional coverage,

$$LR_{uc} = -2\ln[\frac{\phi^{n_1}(1-\phi)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_0}}] \sim \chi^2_{(1)},$$

where ϕ is the parameter of related Bernoulli distribution,which could be $1-\alpha$ or α, n_1 is the number of 1's and n_0 is the number of 0's in the $V_t(\alpha)$ series and $\hat{\pi} = \frac{n_1}{n_1+n_0}$.
2. LR statistic for the test of independence,

$$LR_{ind} = -2\ln[\frac{\hat{\pi}_*^{n_{00}+n_{10}}(1-\hat{\pi}_*)^{n_{11}+n_{01}}}{\hat{\pi}_1^{n_{00}}(1-\hat{\pi}_1)^{n_{01}}\hat{\pi}_2^{n_{11}}(1-\hat{\pi}_2)^{n_{10}}}] \sim \chi^2_{(1)},$$

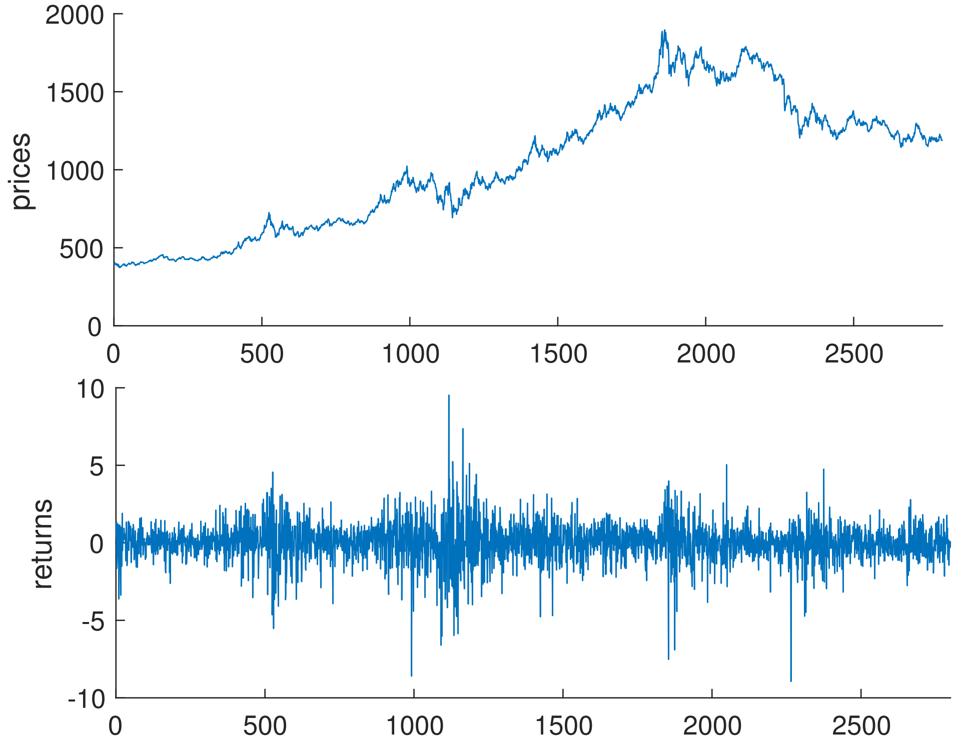

FIGURE 2. Prices and returns of the gold market

where n_{ij} is the number of transition from i to j $(i, j = 0, 1)$ in the $V_t(\alpha)$ series, $\hat{\pi}_1 = \frac{n_{00}}{n_{00}+n_{01}}$, $\hat{\pi}_2 = \frac{n_{11}}{n_{10}+n_{11}}$ and $\hat{\pi}_* = \frac{n_{00}+n_{10}}{n_{00}+n_{01}+n_{10}+n_{11}}$.
3. LR statistic for the test of conditional coverage,

$$LR_{cc} = LR_{ind} + LR_{uc},$$

LR_{cc} has χ^2 distribution with two degrees of freedom. When the value of LR_{cc} is less than the critical value of χ^2 distribution one infer that the conditional coverage is correct and there exist good VaR forecasts.

4. Empirical results

To evaluate the VaR forecasting performance of LST-GARCH with GARCH model for the gold time series, we apply the daily gold prices for the period 14/4/2004 to 13/4/2018, 3460 observation. The first 2800 data are utilized for estimation the parameters and remaining 660 observations are used for forecasting VaR analysis. In Table 1, descriptive statistics of the gold returns are reported. plots the prices and returns of the gold time series. To estimate the parameters, the Bayesian MCMC method, through Gibbs and griddy Gibbs sampling is applied, see [3], and [1]. The estimated parameters of the GARCH and LST-GARCH models and their standard deviations(in parenthesis) are summarized in Table 2. The forecasting results of the VaR tests with common risk levels are reported in Table 3. The second and third columns demonstrate the theoretical expected violations and the number of empirical violations respectively. The last three columns report the statistics for the unconditional coverage(UC), independency(IND) and conditional coverage(CC) tests. From this table, it is notable that the number of

Table 1. Descriptive statistics of the gold price

	Mean	Std. dev.	Skewness	Maximum	Minimum	Kurtosis
Gold data	0.005	1.188	-0.376	9.524	-8.943	9.087

Table 2. Estimation results

	ω	α_1	α_2	β	γ
GARCH	0.345(0.009)	0.413(0.015)	0	0.369(0.012)	0
LST-GARCH	0.292(0.004)	0.602(0.015)	0.188(0.012)	0.218(0.005)	1.055(0.082)

Table 3. VaR results of gold price

Model	α	$E(V_t(\alpha))$	N	UC	IND	CC
	0.99	7	4	1.96	0.049	1.24
	0.975	16.5	8	5.50	0.20	5.7
	0.95	33	15	12.80	0.7	13.50
	0.925	49.5	26	14.34	2.14	16.48
	0.9	66	32	23.47	3.27	26.74
GARCH	0.1	66	43	9.9	1.72	11.68
	0.075	49.50	34	5.80	3.71	9.51
	0.05	33	25	2.19	1.97	4.17
	0.025	17	15	0.14	0.7	0.84
	0.01	7	11	2.48	0.37	2.85
	0.99	7	8	0.28	0.20	0.48
	0.975	16.5	15	0.14	0.70	0.84
	0.95	33	25	2.19	1.97	4.17
	0.925	49.5	46	0.26	6.92	7.18
	0.9	66	61	0.41	6.65	7.06
LST-GARCH	0.1	66	63	0.14	2.21	2.35
	0.075	49.50	42	1.27	1.54	2.80
	0.05	33	35	0.13	3.93	4.07
	0.025	17	22	1.72	1.52	3.24
	0.01	7	15	7.96	0.70	8.67

violations for the LST-GARCH are closer to the expected values than the GARCH model except for two last cases(1% significance and 2.5% level). According to the results of Table 3, at the 5% significance levels, the LR_{uc} test is rejected six times for the GARCH model and only one time for the LST-GARCH model. The LR_{ind} statistic at 5% significance level is bigger than critical value for two cases of LST-GARCH, at 7.5% and 10% levels. The conditional coverage (CC) test is higher than critical value $\chi^2_{0.95}$ with two degrees of freedom five times for the GARCH and three times for LST-GARCH.

5. Conclusion

In financial time series, the positive and negative shocks have different impacts on the market volatility. Indeed conditional variance becomes more volatile by negative shocks relative to positive one. The gold price time series is no exception. One extending of the GARCH model is obtained by considering a convex combination of time dependent logistic weight between the effect of the negative and positive shocks (LST-GARCH).

We fit the GARCH and LST-GARCH models to the gold log returns. The results demonstrate that the LST-GARCH model enables more acceptable one-day-ahead VaR forecasting relative to the GARCH model.

References

1. N. Alemohammad, S. Rezakhah and S.H. Alizadeh, *Markov switching component GARCH model: stability and forecasting*, COMMUN STAT THEOR M. **45** (2016), no. 15, 4332–4348.
2. D. Ardia, *Bayesian estimation of a Markov switching threshold asymmetric GARCH model with Student-t innovations*, Econom J (2009), no. 12, 105–126.

3. L. Bauwens, A. Preminger and V.K. Rombouts, *Theory and inference for Markov switching GARCH model*, Econom J (2010), no. 13, 218–244.
4. T. Bollerslev, *Generalized autoregressive conditional heteroscedasticity*, J Econom (1986), no. 31, 307–327.
5. W.H. Cheng, J.B. Su and Y.P. Tzou, *Value-at-Risk Forecasts in Gold Market Under Oil Shocks*, Middle Eastern Finance and Economics, (2009), no. 4, 48–64.
6. P. Christoffersen, *Evaluating interval forecasting*, Int Econ Rev (1998), no. 39, 841–862.
7. R.F. Engle, *Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation*, Econometrica (1982), no. 50, 987–1007.
8. R.F. Engle, *Discusion: stock volatility and the crash of '87*, Review of Financial Studies (1990), no. 3, 103–106.
9. L.R. Glosten, R. Jagannathan and D. Runkle, *On the relation between the expected value and the volatility of the nominal excess return on stocks*, J Finance (1993), no. 48, 1779–1801.
10. G. Gonzalez-Rivera, *Smooth transition GARCH models*, Stud Nonlinear Dyn Econom (1998), no. 3, 61–78.
11. M. Haas, J. Krause, M.S. Paolella and S.c. Steudc, *Time varying mixture GARCH models and asymmetric volatility*, The North American Journal of Economics and Finance (2013), no. 26, 602–623.
12. M. Lubrano, *Smooth transition GARCH models: a Bayesian approach mixture models*, Recherches Economiques de Louvain (2001), no. 67, 257–287.
13. M.C. Medeiros and A. Veiga, *Modeling multiple regimes in financial volatility with a flexible coefficient GARCH(1,1)*, Econom. Theory (2009), no. 25, 117–161.
14. D.B. Nelson, *Conditional heteroskedasticity in asset returns: a new approach*, Econometrica (1991), no. 59, 347–370.
15. C. Pierdzioch, M. Risse and S. Rohloff, *On the efficiency of the gold market: Results of a real-time forecasting approach*, International Review of Financial Analysis (2014), no. 32, 95–108.
16. S. Truck and K. Liang, *Modelling and forecasting volatility in the gold market*, International Journal of Banking and Finance (2012), no. 9, 48–80.
17. R.S. Tsay, *Analysis of financial time series 2-nd ed*, John Wiley, New York, 2005.
18. J.M. Zakoian, *Threshold heteroskedastic models*, J. Econ. Dyn. Control (1994), no. 27, 577–597.

Department of Mathematics and Computer Science, Shahed University, Tehran, Iran.
E-mail address: n.alemohammad@shahed.ac.ir