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SIMULATION OF FRACTIONAL BROWNIAN MOTION BASING ON ITS SPECTRAL REPRESENTATION

We construct the model of a fractional Brownian motion (fBm) with parameter $\alpha \in (0, 2)$, which approximates such process with given reliability $1 - \delta$, $0 < \delta < 1$, and accuracy $\varepsilon > 0$ in the space $C([0, T])$ basing on a spectral representation of the fBm.

1. Introduction

Simulation of stochastic processes and fields is widely used in many areas of natural and social sciences. There exists a vast literature on the subject, in particular [8,9,11,15, 17,19]. And, naturally, a fractional Brownian motion (fBm) is of special interest because of its applications in physics and financial mathematics. One can find good study of simulation methods for fractional Brownian motion in [1] and [3].

A fractional Brownian motion can be represented in various forms. In the papers [4] and [5] one can find series representations, which can be used for simulation. In particular, series representations of the fBm are used in [6,7,10,13,14,16–19]. In some of these papers there were considered problems of reliability and accuracy of simulation in different functional spaces [6, 7, 13, 14].

Sometimes, it is more convenient and efficient from the implementation point of view to use models of fractional Brownian motion, which are based on its spectral representation, as in [2, 12, 20, 22].

In [12], we presented a method for simulation of fractional Brownian motion with given reliability and accuracy in the space $C([0, T])$, which was based on a spectral representation of the fBm. Now we continue this study and, in this paper, we construct a model of a fractional Brownian motion with parameter $\alpha \in (0, 2)$, which approximates such process with given reliability $1 - \delta$, $0 < \delta < 1$, and accuracy $\varepsilon > 0$ in the space $C([0, T])$ in the case of uniform partition of the simulation interval $[0, \Lambda]$. In Section 2, we define the model basing on the spectral representation of the fBm. Section 3 contains necessary preliminary results, the main theorem with conditions for simulation of a fractional Brownian motion in the space $C([0, T])$, and the corollary for the space $C([0, 1])$. In the last section of the paper we show an example of simulation in the space $C([0, 1])$ in the case of uniform partition of the simulation interval $[0, \Lambda]$ for some values of the process parameter α.

2. A model of fractional Brownian motion

Let (Ω, Σ, P) be a standard probability space and T be a parametric space ($T = [0, T]$ or $T = [0, \infty]$).

Definition 2.1. A random process $\{W_\alpha(t), t \in T\}$ is called fractional Brownian motion with parameter $\alpha \in (0, 2)$, if it is a Gaussian process with zero mean $EW_\alpha(t) = 0$ and

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correlation function

$$R(t,s)=\frac{1}{2}\big(|t|^{\alpha}+|s|^{\alpha}-|t-s|^{\alpha}\big),$$

such that $W_\alpha(0)=0$.

A fractional Brownian motion with parameter $\alpha\in(0,2)$ can be represented in the form of the following stochastic integral [21]:

$$W_\alpha(t)=\frac{A}{\sqrt{\pi}}\bigg(\int_0^\infty \frac{\cos(\lambda t)-1}{\lambda^{\frac{\alpha+1}{2}}}d\xi(\lambda)-\int_0^\infty \frac{\sin(\lambda t)}{\lambda^{\frac{\alpha+1}{2}}}d\eta(\lambda)\bigg),\ t\in[0,T],$$

where $\xi(\lambda),\eta(\lambda)$ are independent real valued standard Wiener processes with

$$E\xi(\lambda)=E\eta(\lambda)=0,\quad E\big(d\xi(\lambda)\big)^2=E\big(d\eta(\lambda)\big)^2=d\lambda,$$

$$A^2=\left\{\frac{2}{\pi}\int_0^\infty\frac{1-\cos(\lambda)}{\lambda^{\alpha+1}}d\lambda\right\}^{-1}=\left\{-\frac{2}{\pi}\Gamma(-\alpha)\cos\Big(\frac{\alpha\pi}{2}\Big)\right\}^{-1}.$$

Let us take an interval $[0,\Lambda]$, $\Lambda>0$, and represent the process $W_\alpha=\{W_\alpha(t),t\in[0,T]\}$ in the form

$$W_\alpha(t)=W_\alpha(t,[0,\epsilon])+W_\alpha(t,[\epsilon,\Lambda])+W_\alpha(t,[\Lambda,\infty]),$$

where $0<\epsilon<\Lambda$ and

$$W_\alpha(t,[a,b])=\frac{A}{\sqrt{\pi}}\bigg(\int_a^b \frac{\cos(\lambda t)-1}{\lambda^{\frac{\alpha+1}{2}}}d\xi(\lambda)-\int_a^b \frac{\sin(\lambda t)}{\lambda^{\frac{\alpha+1}{2}}}d\eta(\lambda)\bigg).$$

Let $0=\lambda_0<\lambda_1<...<\lambda_M=\Lambda$ be a partition of the interval $[0,\Lambda]$, such that $\lambda_1=\epsilon$.

Definition 2.2. We shall define a *model* of the process W_α in the following way:

$$S_M(t,\Lambda)=\frac{A}{\sqrt{\pi}}\bigg(\sum_{i=1}^{M-1}\frac{\cos(\lambda_i t)-1}{\lambda_i^{\frac{\alpha+1}{2}}}(\xi(\lambda_{i+1})-\xi(\lambda_i))-$$

$$-\sum_{i=1}^{M-1}\frac{\sin(\lambda_i t)}{\lambda_i^{\frac{\alpha+1}{2}}}(\eta(\lambda_{i+1})-\eta(\lambda_i))\bigg)=$$

$$=\frac{A}{\sqrt{\pi}}\bigg(\sum_{i=1}^{M-1}\frac{\cos(\lambda_i t)-1}{\lambda_i^{\frac{\alpha+1}{2}}}X_i-\sum_{i=1}^{M-1}\frac{\sin(\lambda_i t)}{\lambda_i^{\frac{\alpha+1}{2}}}Y_i\bigg),\quad t\in[0,T],$$

where $\{X_i,Y_i\},i=1,2,\dots,M-1$, are independent Gaussian random variables with

$$EX_i=EY_i=0,\qquad EX_i^2=EY_i^2=\lambda_{i+1}-\lambda_i.$$

Definition 2.3. The model $S_M=\{S_M(t,\Lambda),t\in[0,T]\}$ approximates the process $W_\alpha=\{W_\alpha(t),t\in[0,T]\}$ with a given reliability $1-\delta$, $0<\delta<1$, and accuracy $\varepsilon>0$ in the space $C([0,T])$ if

$$\mathbf{P}\left\{\sup_{t\in[0,T]}|W_\alpha(t)-S_M(t,\Lambda)|>\varepsilon\right\}\le\delta.$$

3. Simulation of the fBm in the space $C([0,T])$ in the case of uniform partition of the interval $[0,\Lambda]$

In order to get our main results, we shall use the following theorem, which gives us conditions for simulation of the fBm with given reliability and accuracy in the space $C([0,T])$. This theorem was proved in the paper [12] (Theorem 3.1).

Theorem 3.1. *The model S_M approximates the process W_α with a given reliability $1-\delta$, $0<\delta<1$, and accuracy $\varepsilon>0$ in the space $C([0,T])$ if*

$$\gamma_0<\varepsilon, \tag{1}$$

$$\frac{\beta\gamma_0}{K}\le\frac{\varepsilon T^\nu}{2^\nu(\exp\{1/2\}-1)^\nu}, \tag{2}$$

$$2\exp\left\{-\frac{(\varepsilon-\gamma_0)^2}{2\gamma_0^2}\right\}\left(\frac{(\varepsilon-\gamma_0)T^b}{2^b\gamma_0(1-b/\nu)}\left(\frac{\varepsilon K}{\beta\gamma_0}\right)^{\frac{b}{\nu}}+1\right)^{\frac{2}{b}}<\delta, \tag{3}$$

where numbers b and ν are such that $0<b<\nu<\frac{\alpha}{2}$, $\beta=\min\{\gamma_0,\frac{K}{2\nu}\}$,

$$\begin{aligned}\gamma_0 &= \sup_{t\in[0,T]}(\mathsf{E}(X_M(t,\Lambda))^2)^{1/2}=\frac{A}{\sqrt{\pi}}\left(\frac{T^2\lambda_1^{2-\alpha}}{2-\alpha}+\frac{2}{\alpha\Lambda^\alpha}+\right.\\ &\left.+\frac{4T^2}{3}\left(1+\left(\frac{\alpha+1}{2}\right)^2\right)\sum_{i=1}^{M-1}\frac{(\lambda_{i+1}-\lambda_i)^3}{\lambda_i^{\alpha+1}}\right)^{1/2},\end{aligned} \tag{4}$$

$$\begin{aligned}K &= \frac{A\sqrt{3}}{\sqrt{\pi}}\left[\frac{T^{2-2\nu}\lambda_1^{2-\alpha}}{2-\alpha}+\frac{2^{2-2\nu}}{(\alpha-2\nu)\Lambda^{\alpha-2\nu}}+\right.\\ &+2^{4-2\mu}T^{2(\mu-\nu)}\left(\frac{4}{2\mu-\alpha}\sum_{i=1}^{M-1}(\lambda_{i+1}-\lambda_i)^{2\mu-\alpha}+\right.\\ &\left.\left.+\left(\frac{\alpha+1}{2}\right)^2\sum_{i=1}^{M-1}\frac{(\lambda_{i+1}-\lambda_i)^3}{3\lambda_i^{3-(2\mu-\alpha)}}\right)\right]^{\frac{1}{2}},\\ &\mu\in\left(\frac{\alpha}{2};\frac{\alpha+1}{2}\right)\cap(0;1],\end{aligned} \tag{5}$$

$M\in\mathsf{N}$ and $0=\lambda_0<\lambda_1<\ldots<\lambda_M=\Lambda$ is a partition of the interval $[0,\Lambda]$.

In the next theorem we present conditions for simulation of the fBm in the case of uniform partition of the interval $[0,\Lambda]$.

Theorem 3.2. *Let $\lambda_0=0$, $\lambda_i=\frac{i\Lambda}{M}$, $i=1,\ldots,M$. In this case, the model $S_M(t,\Lambda)$ approximates the process W_α in the space $C([0,T])$ with a given accuracy $\varepsilon>0$ and reliability $1-\delta$, $0<\delta<1$, if*

$$\gamma_0^*<\varepsilon, \tag{6}$$

$$\frac{\beta\gamma_0}{K^*}\le\frac{\varepsilon T^\nu}{2^\nu(\exp\{1/2\}-1)^\nu}, \tag{7}$$

$$2\exp\left\{-\frac{(\varepsilon-\gamma_0^*)^2}{2(\gamma_0^*)^2}\right\}\left(\frac{(\varepsilon-\gamma_0^*)T^b}{2^b\gamma_0^*(1-b/\nu)}\left(\frac{\varepsilon K^*}{\beta\gamma_0^*}\right)^{\frac{b}{\nu}}+1\right)^{\frac{2}{b}}<\delta, \tag{8}$$

where numbers b and ν are such that $0 < b < \nu < \frac{\alpha}{2}$, $\beta = \min\{\gamma_0^*, \frac{K^*}{2\nu}\}$,

$$\gamma_0^* = \frac{A}{\sqrt{\pi}}\left(\frac{T^2}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha} + \frac{2}{\alpha\Lambda^\alpha} + \right.$$

$$\left. + \frac{4T^2}{3}\left(1+\frac{1}{\alpha}\right)\left(1+\left(\frac{\alpha+1}{2}\right)^2\right)\left(\frac{\Lambda}{M}\right)^{2-\alpha}\right)^{1/2}, \tag{9}$$

$$K^* = \frac{A\sqrt{3}}{\sqrt{\pi}}\left[\frac{T^{2-2\nu}}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha} + \frac{2^{2-2\nu}}{(\alpha-2\nu)\Lambda^{\alpha-2\nu}} + \right.$$

$$+ 2^{4-2\mu}T^{2(\mu-\nu)}\left(\frac{\Lambda}{M}\right)^{2\mu-\alpha}\left(\frac{4(M-1)}{2\mu-\alpha} + \right. \tag{10}$$

$$\left.\left. + \frac{1}{3}\left(\frac{\alpha+1}{2}\right)^2\left(1+\frac{1}{2-(2\mu-\alpha)}\right)\right)\right]^{\frac{1}{2}},$$

$$\mu \in \left(\frac{\alpha}{2}; \frac{\alpha+1}{2}\right) \cap (0;1].$$

Proof. In the case of uniform partition of the interval $[0,\Lambda]$, that is, if $\lambda_i = \frac{i\Lambda}{M}$, $i = \overline{0,M}$, and, hence, $\triangle\lambda = \lambda_{i+1} - \lambda_i = \frac{\Lambda}{M}$, $i = 0, \ldots, M-1$, we can estimate the constants γ_0 and K in the assertion of the Theorem 3.1 in the following way.

$$\gamma_0 = \frac{A}{\sqrt{\pi}}\left(\frac{T^2\lambda_1^{2-\alpha}}{2-\alpha} + \frac{2}{\alpha\Lambda^\alpha} + \right.$$

$$\left. + \frac{4T^2}{3}\left(1+\left(\frac{\alpha+1}{2}\right)^2\right)\sum_{i=1}^{M-1}\frac{(\lambda_{i+1}-\lambda_i)^3}{\lambda_i^{\alpha+1}}\right)^{1/2} =$$

$$= \frac{A}{\sqrt{\pi}}\left(\frac{T^2}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha} + \frac{2}{\alpha\Lambda^\alpha} + \right.$$

$$\left. + \frac{4T^2}{3}\left(1+\left(\frac{\alpha+1}{2}\right)^2\right)\sum_{i=1}^{M-1}\frac{\left(\frac{\Lambda}{M}\right)^3}{\left(\frac{i\Lambda}{M}\right)^{\alpha+1}}\right)^{1/2} =$$

$$= \frac{A}{\sqrt{\pi}}\left(\frac{T^2}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha} + \frac{2}{\alpha\Lambda^\alpha} + \right.$$

$$\left. + \frac{4T^2}{3}\left(1+\left(\frac{\alpha+1}{2}\right)^2\right)\left(\frac{\Lambda}{M}\right)^{2-\alpha}\sum_{i=1}^{M-1}\frac{1}{i^{\alpha+1}}\right)^{1/2}.$$

Let us estimate the sum $\sum\limits_{i=1}^{M-1}\frac{1}{i^{\alpha+1}}$ in the expression above:

$$\sum_{i=1}^{M-1}\frac{1}{i^{\alpha+1}} = 1 + \sum_{i=2}^{M-1}\frac{1}{i^{\alpha+1}} \le 1 + \sum_{i=2}^{M-1}\int_{i-1}^{i}\frac{1}{u^{\alpha+1}}\mathrm{d}u =$$

$$= 1 + \int_{1}^{M-1}\frac{1}{u^{\alpha+1}}\mathrm{d}u = 1 - \frac{1}{\alpha(M-1)^\alpha} + \frac{1}{\alpha} \le 1 + \frac{1}{\alpha}$$

Thus, for the γ_0 we get the following estimate:

$$\begin{aligned}\gamma_0 &\le \frac{A}{\sqrt{\pi}}\Bigg(\frac{T^2}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha}+\frac{2}{\alpha\Lambda^\alpha}+\\ &+\frac{4T^2}{3}\left(1+\frac{1}{\alpha}\right)\left(1+\left(\frac{\alpha+1}{2}\right)^2\right)\left(\frac{\Lambda}{M}\right)^{2-\alpha}\Bigg)^{1/2}=:\gamma_0^*.\end{aligned}$$

Now we shall consider the constant K:

$$\begin{aligned}K &= \frac{A\sqrt{3}}{\sqrt{\pi}}\Bigg[\frac{T^{2-2\nu}\lambda_1^{2-\alpha}}{2-\alpha}+\frac{2^{2-2\nu}}{(\alpha-2\nu)\Lambda^{\alpha-2\nu}}+\\ &+ 2^{4-2\mu}T^{2(\mu-\nu)}\Bigg(\frac{4}{2\mu-\alpha}\sum_{i=1}^{M-1}(\lambda_{i+1}-\lambda_i)^{2\mu-\alpha}+\\ &+\left(\frac{\alpha+1}{2}\right)^2\sum_{i=1}^{M-1}\frac{(\lambda_{i+1}-\lambda_i)^3}{3\lambda_i^{3-(2\mu-\alpha)}}\Bigg)\Bigg]^{\frac{1}{2}}=\end{aligned}$$

$$\begin{aligned}&= \frac{A\sqrt{3}}{\sqrt{\pi}}\Bigg[\frac{T^{2-2\nu}}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha}+\frac{2^{2-2\nu}}{(\alpha-2\nu)\Lambda^{\alpha-2\nu}}+\\ &+ 2^{4-2\mu}T^{2(\mu-\nu)}\Bigg(\frac{4}{2\mu-\alpha}\sum_{i=1}^{M-1}\left(\frac{\Lambda}{M}\right)^{2\mu-\alpha}+\\ &+\left(\frac{\alpha+1}{2}\right)^2\sum_{i=1}^{M-1}\frac{\left(\frac{\Lambda}{M}\right)^3}{3\left(\frac{i\Lambda}{M}\right)^{3-(2\mu-\alpha)}}\Bigg)\Bigg]^{\frac{1}{2}}=\\ &= \frac{A\sqrt{3}}{\sqrt{\pi}}\Bigg[\frac{T^{2-2\nu}}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha}+\frac{2^{2-2\nu}}{(\alpha-2\nu)\Lambda^{\alpha-2\nu}}+\\ &+ 2^{4-2\mu}T^{2(\mu-\nu)}\Bigg(\frac{4(M-1)}{2\mu-\alpha}\left(\frac{\Lambda}{M}\right)^{2\mu-\alpha}+\\ &+\frac{1}{3}\left(\frac{\alpha+1}{2}\right)^2\left(\frac{\Lambda}{M}\right)^{2\mu-\alpha}\sum_{i=1}^{M-1}\frac{1}{i^{3-(2\mu-\alpha)}}\Bigg)\Bigg]^{\frac{1}{2}}\end{aligned}$$

For the sum $\sum_{i=1}^{M-1}\frac{1}{i^{3-(2\mu-\alpha)}}$ we get the next estimate:

$$\sum_{i=1}^{M-1}\frac{1}{i^{3-(2\mu-\alpha)}}=1+\sum_{i=2}^{M-1}\frac{1}{i^{3-(2\mu-\alpha)}}\le 1+\sum_{i=2}^{M-1}\int_{i-1}^{i}\frac{1}{u^{3-(2\mu-\alpha)}}\mathrm{d}u=$$

$$1+\int_1^{M-1}\frac{1}{u^{3-(2\mu-\alpha)}}\mathrm{d}u=1-\frac{1}{2-(2\mu-\alpha)}\left(\frac{1}{(M-1)^{2-(2\mu-\alpha)}}-1\right)\le 1+\frac{1}{2-(2\mu-\alpha)}$$

Thus, for the K we have:

$$\begin{aligned} K &\leq \frac{A\sqrt{3}}{\sqrt{\pi}}\left[\frac{T^{2-2\nu}}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha} + \frac{2^{2-2\nu}}{(\alpha-2\nu)\Lambda^{\alpha-2\nu}} + \right. \\ &+ 2^{4-2\mu}T^{2(\mu-\nu)}\left(\frac{\Lambda}{M}\right)^{2\mu-\alpha}\left(\frac{4(M-1)}{2\mu-\alpha} + \right. \\ &+ \left.\left.\frac{1}{3}\left(\frac{\alpha+1}{2}\right)^2\left(1+\frac{1}{2-(2\mu-\alpha)}\right)\right)\right]^{\frac{1}{2}} =: K^* \end{aligned}$$

Now, if for the obtained constants γ_0^* and K^* the conditions (6)–(8) of Theorem 3.2 hold, then the conditions (1)–(3) of Theorem 3.1 hold correspondingly, and the model $S_M(t,\Lambda)$ approximates the process W_α in the space $C([0,T])$ with a given accuracy $\varepsilon > 0$ and reliability $1-\delta$, $0<\delta<1$. □

Corollary 3.1. *In case of $T = 1$ from Theorem 3.2 follows, that the model S_M approximates the process W_α in the space $C([0,1])$ with a given accuracy $\varepsilon > 0$ and reliability $1-\delta$, $0<\delta<1$, if*

$$\hat{\gamma}_0^* < \varepsilon, \tag{11}$$

$$\frac{\beta\hat{\gamma}_0^*}{\hat{K}^*} \leq \frac{\varepsilon}{2^\nu(\exp\{1/2\}-1)^\nu}, \tag{12}$$

$$2\exp\left\{-\frac{(\varepsilon-\hat{\gamma}_0^*)^2}{2(\hat{\gamma}_0^*)^2}\right\}\left(\frac{\varepsilon-\hat{\gamma}_0^*}{2^b\hat{\gamma}_0^*(1-b/\nu)}\left(\frac{\varepsilon\hat{K}^*}{\beta\hat{\gamma}_0^*}\right)^{\frac{b}{\nu}}+1\right)^{\frac{2}{b}} < \delta, \tag{13}$$

where numbers b and ν are such that $0<b<\nu<\frac{\alpha}{2}$, $\beta = \min\{\hat{\gamma}_0^*, \frac{\hat{K}^*}{2\nu}\}$,

$$\begin{aligned} \hat{\gamma}_0^* &= \frac{A}{\sqrt{\pi}}\left(\frac{1}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha} + \frac{2}{\alpha\Lambda^\alpha} + \right. \\ &+ \left.\frac{4}{3}\left(1+\frac{1}{\alpha}\right)\left(1+\left(\frac{\alpha+1}{2}\right)^2\right)\left(\frac{\Lambda}{M}\right)^{2-\alpha}\right)^{1/2}, \end{aligned} \tag{14}$$

$$\begin{aligned} \hat{K}^* &= \frac{A\sqrt{3}}{\sqrt{\pi}}\left[\frac{1}{2-\alpha}\left(\frac{\Lambda}{M}\right)^{2-\alpha} + \frac{2^{2-2\nu}}{(\alpha-2\nu)\Lambda^{\alpha-2\nu}} + \right. \\ &+ 2^{4-2\mu}\left(\frac{\Lambda}{M}\right)^{2\mu-\alpha}\left(\frac{4(M-1)}{2\mu-\alpha} + \right. \\ &+ \left.\left.\frac{1}{3}\left(\frac{\alpha+1}{2}\right)^2\left(1+\frac{1}{2-(2\mu-\alpha)}\right)\right)\right]^{\frac{1}{2}}, \\ &\mu \in \left(\frac{\alpha}{2}; \frac{\alpha+1}{2}\right) \cap (0;1]. \end{aligned} \tag{15}$$

In order to get better results, it is necessary to optimize model parameters. In the next corollary, we intend to reach the minimal value of $\hat{\gamma}_0^*$.

Corollary 3.2. *It is easy to see that $\hat{\gamma}_0^*$ will be minimal, if*

$$\Lambda = M^{\frac{2-\alpha}{2}}C_\alpha,$$

where

$$C_\alpha = \left(\frac{6\alpha}{3\alpha + (2-\alpha)(\alpha+1)(4+(\alpha+1)^2)}\right)^{1/2} = \left(\frac{2}{1+(2-\alpha)B(\alpha)}\right)^{1/2},$$

$$B(\alpha) = \frac{4}{3}\left(1+\frac{1}{\alpha}\right)\left(1+\left(\frac{\alpha+1}{2}\right)^2\right).$$

In this case we get that

$$(\hat{\gamma}_0^*)^2 = \frac{A^2}{\pi}\frac{D(\alpha)}{M^{\frac{\alpha(2-\alpha)}{2}}},$$

where

$$D(\alpha) = \frac{2^{2-\frac{\alpha}{2}}(1+(2-\alpha)B(\alpha))^{1/2}}{\alpha(2-\alpha)},$$

and

$$\begin{aligned} \hat{K}^* &= \frac{A\sqrt{3}}{\sqrt{\pi}}\left[\frac{(C_\alpha)^{2-\alpha}}{2-\alpha} + \frac{2^{2-2\nu}M^{(2-\alpha)\nu}}{(C_\alpha)^{\alpha-2\nu}(\alpha-2\nu)}\right. \\ &+ \left.(C_\alpha)^{2\mu-\alpha}2^{4-2\mu}G_M(\alpha)M^{\alpha(1-\mu)}\right]^{1/2}, \end{aligned}$$

where

$$G_M(\alpha) = \left[\frac{4(M-1)}{2\mu-\alpha} + \frac{1}{3}\left(\frac{\alpha+1}{2}\right)^2\left(1+\frac{1}{2-(2\mu-\alpha)}\right)\right].$$

4. Numerical results

Basing on the results of the Corollary 3.2, we have estimated parameters Λ and M in the case of uniform partition of the interval $[0,\Lambda]$, reliability $1-\delta = 0.95$, accuracy $\varepsilon = 0.1$ and technical model parameters $\nu = 0.499\alpha$, $b = 0.8\nu$. Some of the results obtained are presented in Table 1. For comparison, in Table 2 we show numerical values of the model parameters calculated basing on our results published in paper [12]. The trajectories of models of the fBm for some values of the parameter α are shown on Figures 1–4.

Table 1. Model parameters for some values of α

α	Λ	M
1.0	25130	2×10^9
1.1	8988	2×10^9
1.3	1960	9×10^9
1.5	703	9×10^{11}

Table 2. Model parameters basing on the results of paper [12]

α	Λ	M
1.0	25000	3.3×10^9
1.1	23500	5.4×10^9
1.3	2100	3.3×10^{10}
1.5	595	7.0×10^{12}

Conclusions

We have presented a model of a fractional Brownian motion with parameter $\alpha \in (0,2)$, which approximates such process with given reliability $1-\delta$, $0<\delta<1$, and accuracy $\varepsilon > 0$ in the space $C([0,T])$. The model is based on a spectral representation of the fBm. In the case of uniform partition of the simulation interval $[0,\Lambda]$, the results obtained are better and more convenient for practical use, than in our previous paper on this subject [12].

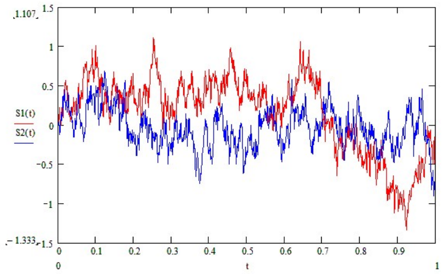

FIGURE 1. Models for fBm with $\alpha = 0.6$

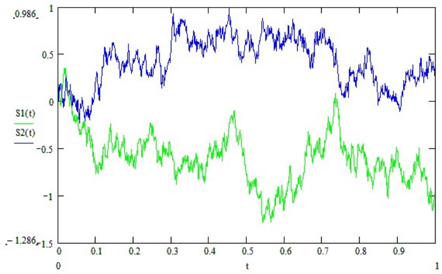

FIGURE 2. Models for fBm with $\alpha = 0.8$

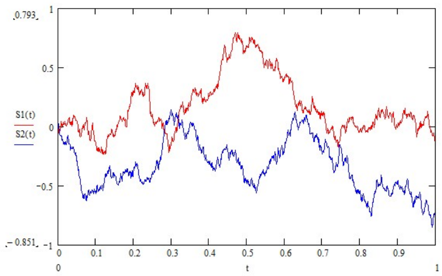

FIGURE 3. Models for fBm with $\alpha = 1.1$

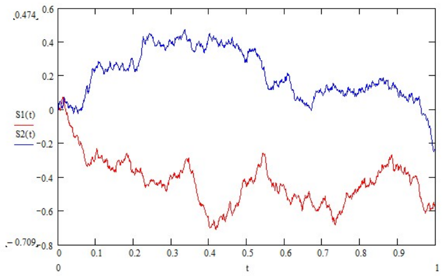

FIGURE 4. Models for fBm with $\alpha = 1.3$

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