MYKOLA PORTENKO

A 100th ANNIVERSARY TRIBUTE TO IOSIF IL'ICH GIKHMAN

Iosif Il'ich Gikhman (May 26, 1918 – July 30, 1985)

This note is written in commemoration of I. I. Gikhman, who was not only an outstanding mathematician and excellent teacher, but was also one of the founders of the Ukrainian probabilistic school. Because his scientific achievements were described in details by many authors (see articles [7]–[13]), in my note I will only briefly touch upon his main scientific success, which is, of course, creating the theory of stochastic differential equations (see his articles [1]–[3] published during the 1940s and the early 1950s). His version of this theory is less popular than that by K. Itô, created independently around the same time (see [4]–[6]). I do not think it would be out of place to renew some of the peculiarities of these theories here.

With the influence of I. I. Gikhman, A. V. Skorokhod was engaged in research on the theory of stochastic differential equations at the beginning of 1957, and by the end of 1961 he had published his first book devoted mainly to that theory. The book turned out to be a very important stage in the evolution of the theory, because of the new ideas and methods introduced by A. V. Skorokhod[1]. From the 1960s to the early 1980s, I. I. Gikhman collaborated with A. V. Skorokhod to produce fruitful work, in particular, on the theory of stochastic differential equations. They jointly published two books on the topic and even drew up huge plans for further books after publishing the 1982 book. Those plans, however, were not carried out because of a serious illness of I. I. Gikhman and his death.

At the end of this note I will present my point of view on the formation of the Ukrainian school of probability theory and the role of I. I. Gikhman in that process.

To complete the introductory part of my note, I briefly outline my relations with I. I. Gikhman. During my student years (1958-1963) at Kyiv University, I took many of his basic and special courses in mathematics, and under his supervision I carried on my postgraduate studies. When he and his family moved from Kyiv to Donetsk in 1966, I followed suit, and worked there under his guidance until my return back to Kyiv in 1972, since which time I have worked at the Institute of mathematics in Kyiv. We kept in touch after I left Donetsk, saw each other often, and even had some businesses in common, in particular, the ones connected with publishing volumes of the journal "Theory of Stochastic Processes" (this journal was founded by him in 1973). He was one of the opponents of my Doctoral dissertation (Kyiv 1978). We were also both members of the World congress of mathematicians in Warsaw in 1983. At the beginning of August 1985, I was in Donetsk among the people gathered to see him off on his last journey.

ON THE ORIGIN OF THE THEORY OF STOCHASTIC DIFFERENTIAL EQUATIONS

K. Itô developed the theory of stochastic differential equations based upon the notion of a stochastic integral he had introduced. His notion was a generalization of Wiener's in two directions: firstly, his integrand was a random function (Wiener constructed the integral of a non-random one) and, secondly, he constructed integrals not only with respect to Brownian motion but also with respect to a (centered) Poisson measure. Given some local characteristics of a stochastic process to be constructed and, in addition, such "simple" objects as Brownian motion and a Poisson measure, he wrote down a stochastic integral equation (it could be written as a differential one) whose solution gave the trajectories of the process desired. Under some conditions on given coefficients (the local characteristics mentioned above), he managed to prove the existence and uniqueness of the solution and to establish it as a Markov process. The set of stochastic processes that were differentiable in Itô's sense was endowed by a calculus different from the classical type (for example, the Itô formula meant a new rule of differentiating a function of a stochastic process having Itô's stochastic differential). The approach to the theory of stochastic differential equations given by K. Itô turned out to be exceptionally proper: in most of the monographs on the topics, the basic notion is the Itô stochastic integral or some of its generalizations.

---

[1]I have already had an opportunity to describe my impressions about that book in my article "A few words about the first book by A. V. Skorokhod" published in "Newsletter of the European Mathematical Society", No. 94, December 2014, pp. 25-29. Some part of that article is devoted to I. I. Gikhman and it is used here, in this article. I should say that for a Ukrainian probabilist of my generation, it is impossible to narrate about the one of the famous duet Gikhman–Skorokhod without any mention of the other one.

I. I. Gikhman did not have such a notion. Nevertheless, his definition of a stochastic differential equation was quite rigorous in a mathematical sense. It was based on a given field of random processes that locally determined the increments of the process to be constructed as a solution to the corresponding stochastic differential equation. Under some conditions on a given field, I. I. Gikhman proved the theorem on the existence and uniqueness of the solution with given initial data. If the random field did not possess a property of after-effect then the solution was a Markov process. In the case of that field being given by a vector field of macroscopic velocities plus the increments of Brownian motion transformed by a given operator field, the corresponding solution turned out to be a differentiable function with respect to the initial data (under the assumption, of course, that the mentioned fields were given by smooth functions in spatial arguments). With this result, I. I. Gikhman managed to prove the theorem on the existence of a solution to Kolmogorov's backward equation (i.e. a second-order partial differential equation of parabolic type) without any assumptions on the non-degeneracy property of the matrix consisting of the coefficients of the second spatial derivatives (it is well known how important such a property is in the analytical theory of those equations). That was a significant result showing that some theorems in the theory of partial differential equations can be proved with the use of purely probabilistic methods.

I consider it one more significant I. I. Gikhman's success that under his influence A. V. Skorokhod was engaged in investigations in the theory of stochastic differential equations. A. V. Skorokhod started lecturing at Kyiv University in 1957. Over the three previous years, he was a postgraduate student at Moscow University. His studies there were a dazzling success: he had formulated and proved the most general limit theorems for stochastic processes and, moreover, he had invented an original method for proving them. In spite of his young age, he had already succeeded in gaining authority amongst experts in probability theory. I think that he had the opportunity to acquaint himself with Itô theory during his studies in Moscow. As far as I know, just after coming back to Kyiv, his regular discussions with I. I. Gikhman started taking place and he was able to comprehend Gikhman's approach to the theory. At the end of 1961 in the publishing house of Kyiv University, the book "Studies in the theory of random processes" by A. V. Skorokhod came out. Besides the title, there was a subtitle on its cover, namely "Stochastic differential equations and limit theorems for Markov processes". The assertions expounded in that book, as well as the methods used by A. V. Skorokhod for proving them, were fundamentally different from those that were typical in the theory of stochastic differential equations at that time: the book was full of new ideas, new methods and new results. A few years later, in 1968, I. I. Gikhman and A. V. Skorokhod published the book "Stochastic differential equations" in the publishing house "Naukova dumka" and in 1982 they published a new book with close title "Stochastic differential equations and their applications" in the same publishing house. Here are the words of A. V. Skorokhod about his joint plans with I. I. Gikhman after publishing the 1982 book: "We decided to write a new book that would bring together material relating to applied areas in the theory of stochastic equations. We intended to treat equations in infinite-dimensional spaces, in particular, infinite systems of stochastic equations; the theory of linear equations in infinite-dimensional spaces and the semigroups connected with them, in particular, stochastic partial differential equations of evolution type; equations for conditionally Markov processes and the equations of nonlinear filtration connected with them; and the asymptotic behavior of solutions of stochastic equations, including ergodic theory, the method of averaging, and the theory of stability. The plan of the book was discussed for a fairly long time, and we convinced ourselves at last that it was impossible to present all these topics in a single book. We then decided to treat the last topic. This choice was made under the influence of the interests of Iosif Il'ich, who, as a student of

N. N. Bogolyubov, had directed much attention to the study of the asymptotic behavior of systems undergoing random perturbations." These were the plans, but . . . .

In 1987 A. V. Skorokhod published the book "Asymptotic methods in the theory of stochastic differential equations" in "Naukova dumka". Skorokhod's words cited above, as well as the following ones are taken from Foreword to that book. "While recognizing how far this book was from what we had envisioned, I wrote it nevertheless, hoping at least by the choice of topic to pay homage to the shining memory of my teacher and friend."

ON THE FORMATION OF THE UKRAINIAN SCHOOL OF PROBABILITY THEORY

The school of probability theory in Ukraine was formed in the 1940s and the early 1950s. As I see it, one should distinguish between the two quite different lines in it.

The first one was connected with B. V. Gnedenko, who left Moscow for Ukraine in 1945 (he first came to Lviv and then moved to Kyiv in 1949). With his appearance in Ukraine, traditions of the famous Moscow probabilistic school with academician A. N. Kolmogorov at its head were brought into the circle of young Ukrainian mathematicians. I should like to emphasize that it was B. V. Gnedenko who in 1953 proposed to A. V. Skorokhod (he had just graduated from Kyiv University) the problem of extending the Donsker's invariance principle on the situation when the limiting process for the sums of independent random variables was a general process with independent increments (not necessarily continuous, as was in Donsker's result). I have already mentioned above that A. V. Skorokhod formulated and proved the general invariance principle in an accomplished form during his postgraduate studies at Moscow University.

The second line in the Ukrainian probabilistic school was originated by I. I. Gikhman. He came to probability theory from the school of N. N. Bogolyubov, that is, from the school of mathematical physics. One can recognize traditions of that school in I. I. Gikhman's approach to the theory of stochastic differential equations. His little note [1] gave start to the development of that theory in Ukraine.

It is clear that A. V. Skorokhod was a crucial point on each of those lines. I think that in the formation of the Ukrainian probabilistic school each of these three outstanding mathematicians played the role that cannot be over-estimated.

References

1. I. I. Gikhman, *On a scheme for formation of random processes*, Doklady Akad. Nauk USSR **58** (1947), 961-964. (Russian)
2. I. I. Gikhman, *On some differential equations with random functions*, Ukrain. Mat. Zh. **2** (1950), no. 3, 45-69. (Russian)
3. I. I. Gikhman, *On the theory of differential equations of random processes. I, II*, Ukrain. Mat. Zh. **2** (1950), no. 4, 37-63; **3** (1951), 317-339. (Russian)
4. K. Itô, *Stochastic integral*, Proc. Imp. Acad., Tokyo, **20** (1944), 519-524.
5. K. Itô, *On a stochastic differential equation*, Proc. Jap. Acad., Tokyo, **22** (1946), 32-35.
6. K. Itô, *On stochastic differential equations*, Mem. Am. Math. Soc., **4** (1951), 1-51.
7. I. I. Danilyuk, V. S. Korolyuk, Yu. A. Mitropol'skii, A. V. Skorokhod and M. I. Yadrenko, *Iosif Il'ich Gikhman (on the occasion of his 50th birthday)*, Ukrain. Mat. Zh. **20** (1968), 363-366. (Russian)
8. V. S. Korolyuk, Yu. A. Mitropol'skii, A. V. Skorokhod, *Iosif Il'ich Gikhman (on the occasion of his 60th birthday)*, Uspekhi Mat. Nauk **33** (1978), no. 5(203), 205-208. (Russian)
9. *Iosif Il'ich Gikhman*, Teor. Veroyatnost. I Primen. **31** (1986), 126-127. (Russian)
10. A. V. Skorokhod and M. I. Yadrenko, *I. I. Gikhman: a brief review of scientific and pedagogic activity*, Teor. Imovirnost. ta Mat. Statist. **50** (1994), 1-5. (Ukrainian)
11. V. Korolyuk, Yu. Lin'kov, N. Portenko and A. Shatashvili, *The teacher*, Obshchestvennyj Nauchno-Proizvodsvennyj Zhurnal "Vsyo" (1998), no. 4, 52-54. (Russian)

12. Yurii N. Lin'kov and Mikhail I. Yadrenko, *To the 80th birthday of Iosif I. Gikhman*, Theory of Stochastic Processes **4(20)** (1998), no. 1-2, 1-16.
13. M. I. Portenko, *To the 90th birthday of I. I. Gikhman*, Mathematics Today '08, **14** (2008), 159-166. (Ukrainian)

INSTITUTE OF MATHEMATICS OF UKRAINIAN NATIONAL ACADEMY OF SCIENCES
Current address: 3, Tereschenkivska str., 01601, Kyiv-4, Ukraine
E-mail address: `portenko@imath.kiev.ua`