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MEASURES OF FINANCIAL RISKS AND MARKET CRASHES

The problem of particular importance in financial risk management is forecasting the magnitude of a market crash. We address this problem using statistical inference on heavy–tailed distributions. Our approach involves accurate estimates of the tail index, extreme quantiles, and the mean excess function. We apply our approach to real financial data, and argue that the September 2001 crash had two components: one (systematic) could be predicted, while another (non–systematic) was due to the shock of the event. We present empirical evidence that the degree of tail heaviness can change considerably as one switches to less frequent data. This fact has important implications to the problem of estimating financial risks.

1. Introduction

The classical mean–variance portfolio theory utilizes the idea of balancing the expected return vs. the risk represented by the standard deviation. The use of the standard deviation as a measures of risk is justified if portfolio returns are light–tailed. In reality, financial data often appear heavy–tailed. The evidence of heavy tails was accounted as early as in 1960s (see Mandelbrot (1963) and Fama & Roll (1968)). This fact is nowadays the subject of textbooks, see, e.g., Luenberger [12], p. 302, and Embrechts et al. (1997), p. 404–405. The feature is particularly common to "frequent" data, e.g., daily log–returns of stock prices, while log–returns of less frequent data can exhibits lighter tails, well in line with the central limit theorem.

If data is heavy–tailed, then the standard deviation is no longer responsible for extreme movements of portfolio returns even if the portfolio is optimal in the sense of the mean–variance theory, and can hardly be considered a proper measure of risk. Recall that if data is heavy–tailed then a single sample element, e.g., the loss over one particular day, can make

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a major contribution to the total loss over a considerable period of time. For instance, on "black Monday" 19.10.1987 the S&P500 index fell by 20.5% erasing all the index had gained since March 1986. Sometimes data exhibits such heavy tails that the variance is likely to be infinite. Value–at–Risk and Expected Shortfall appear more suitable measures of risk.

Value–at–Risk (VaR) was popularized as a measure of financial risks by JP Morgan in 1990s. It indicates how much money a bank should put aside in order to offset the risk of an unfavorable market movement. If one deals with daily data, then 1%–VaR shows how far the value of a portfolio can fall once in approximately 100 days. Another measure of risk, Expected Shortfall (ES), presents the average loss given there is a fall beyond VaR. Many banks routinely calculate VaR in order to monitor the current exposure of their portfolios to market risks. For instance, Goldman Sachs uses 5%–VaR; Citigroup, Credit Suisse First Boston, Deutsche Bank, JP Morgan Chase and Morgan Stanley use 1%–VaR.

The important practical question is how to evaluate VaR and ES. If data followed the normal distribution, then VaR would be a constant times the standard deviation. In reality, daily and weekly rates of return of many stocks and stock indexes appear heavy–tailed, ruling out the assumption of normality.

The problem of reliable estimation of VaR and ES in the presence of heavy tails is demanding, it remained open for a long while. A reliable procedure of practical estimation of VaR and ES from (possibly dependent) heavy–tailed data was introduced in Novak (2002). The approach was tested on samples of simulated data where the true values of the tail index, VaR, and ES are known. An application of the procedure to the problem of predicting the magnitude of the "black Monday" crash in Novak & Beirlant (2006) confirmed the accuracy of the approach in comparison with those of McNeil (1998) and Matthys & Beirlant (2001).

While the market crash in October 1987 was purely "market driven", the collapse of S&P500 after September the 11th was obviously triggered by the tragic event in New York. We argue in this paper that the 2001 crash had two components: one (systematic) could be predicted using the method presented below, while another (non–systematic) was due to the shock of the event.

We show also that the degree of heaviness of the data tails can change considerably as one switches to less frequent data. This fact has important implications in risk management: it means that the textbook rule for the evaluation of VaR over a period of time using daily VaR estimates multiplied by the square root of the number of days (cf. Jorion (2001), Ho & Lee (2004), p. 533, Tapiero (2004), p. 313) can be misleading.

The next two sections are devoted to the problems of statistical inference on dependent heavy–tailed data. We introduce a new estimator of the tail index and establish its consistency. In section 4 we apply our approach to real data and show what a forecast of the magnitude of possible losses one could make on the eve of September the 11th. We present empirical evidence that more frequent financial data has considerably heavier tails. This fact contradicts to the theory, and has important implications to the problem of risk evaluation.

2. Heavy tails

Which tails should be considered heavy? Following Resnick (1997), we say that the distribution has a *heavy tail* if

$$F(x) := \mathbb{P}(X \leq x) = L(x)|x|^{-\alpha} \qquad (\alpha > 0), \tag{1}$$

where the (unknown) function L is slowly varying: $\lim_{x\to-\infty} L(xt)/L(x) = 1$ for all $t > 0$. Here X stands for a sample element, say, daily log–return of a stock price. The definition is given for the left tail; it can be obviously reformulated for the right tail:

$$\mathbb{P}(X > x) = L(x)x^{-1/a} \qquad (a > 0), \tag{1*}$$

where the (unknown) function L is slowly varying at infinity.

The number α in (1) or (1*) is called the *tail index*. It is the main characteristic describing the tail behavior. The smaller is α, the heavier is the tail as well as the likelihood of extreme movements. Thus, the tail index appears a proper measure of the "degree of heaviness" of the tail of a distribution.

Distributions that obey (1) form a *non–parametric* (semi–parametric) family of probability laws. It includes Cauchy, Student's, Pareto and ARCH distributions, among others. The non–parametric setup is the advantage of our approach since there are doubts that parametric models accurately describe real financial data (cf. Capobianco (2002), Section 4.3). The problem with the parametric approach is that we never know if the unknown distribution belongs to a chosen parametric family (hypothesis H_0); "parametric" papers usually do not tell readers what authors' findings are worth if the unknown distribution does not come from a chosen parametric family. Another problem with the parametric approach is the lack of robustness. For instance, McNeil (1997, Section 4.6) indicates that parametric tail index and VaR estimators can change by 20% if one sample element is removed. The advantage of the non–parametric setup is that a chosen class of distributions is so rich that the problem of testing H_0 does not arise.

There is a number of procedures to check if the tail is heavy. One possibility is to use a qq–plot (quantile–quantile–plot). Recall that the *quantile* F^{-1} is the inverse to the distribution function F:

$$F^{-1}(y) = \inf\{t : F(t) \geq y\} \qquad (y \in \mathbb{R}).$$

One can plot empirical quantiles versus quantiles of a given distribution function F_o. If the line is approximately linear, then tails of F and F_o are close up to a linear transform, cf. Embrechts et al. (1997), p. 292–293, and Mikosch (2004), p. 90–91. Another procedure is based on the mean excess function, see Embrechts et al. (1997), p. 355, Novak (2002) and Mikosch (2004). For instance, the analysis in Matthys & Beirlant (2001) confirms that daily log–returns of the S&P500 index are heavy–tailed.

Up to a sign, VaR is the extreme quantile:

$$m\%\text{–VaR} = -F^{-1}(m/100).$$

For instance, if 1%–VaR equals 0.04, then in 1% worst cases X can fall below –0.04. ES is the *average fall beyond* VaR: ES $= \mathbb{E}\{y - X|X \le y\}$, where $y = -\text{VaR}$.

Below we prefer to deal with positive numbers; in particular, we speak about "20.5% fall" of the S&P500 index on the "black Monday" instead of "–20.5% rate of return", and switch from X to $-X$, where X stands for a sample element. We assume (1*); q–VaR (denoted in the sequel by y_q) is now defined as the upper quantile of level q (the inverse of $\bar{F} = 1 - F$):

$$y_q = \bar{F}^{-1}(q); \tag{2}$$

ES is now the average excess over the VaR: if y =VaR, then ES is

$$E(y) = \mathbb{E}\{X - y|X > y\}\,.$$

It is well defined if the tail index $\alpha > 1$. Novak (2002) points out that $E(y) \sim y/(\alpha - 1)$ as $y \to \infty$ if (1*) holds. One can accurately estimate ES if consistent estimators of α and VaR are available and α is not close to 1 (the difficulty of ES estimation when α is close to 1 is pointed out by Yamai & Yoshiba (2002)). The sum VaR+ES $= \mathbb{E}\{X|X > y_q\}$ is the average value of X given X exceeds q–VaR; it is a "typical" value of a non–typical (extreme) observation.

When one estimates quantiles of level q with q bounded away from 0 and 1, the *empirical quantile estimator* (the inverse of the empirical distribution function) is the most natural choice. Simulation study in Novak (2002) indicates that the empirical quantile estimator works poorly when one estimates *extreme* quantiles (quantiles of level $q \le 0.05$). This observation is in line with the theory: if q is "small" (close to 0), then the empirical inference is based on very few (if not none) elements of a sample. By contrast, our approach is based on observations of "moderate magnitude" and hence is much more robust.

3. Inference on heavy tails

It is widely accepted that financial data is typically dependent. One can measure dependence using, for instance, mixing (week–dependence) coefficients φ and ρ (the definition of mixing coefficients can be found, e.g., in Embrechts et al. (1997) and Novak (2005)). The fact that past data has little effect on today's price movements can be formalised by the equation

$$\lim_{k\to 0} \varphi(k) = 0 \tag{3}$$

or

$$\sum_{k\geq 1} k^{-1}\rho(k) < \infty\,. \tag{4}$$

The latter holds, e.g., if $\rho(k) \asymp (\ln k)^{-c}$ $(c > 1)$. Note that $\rho(k) \leq \varphi^{1/2}(k)$.

We assume in the sequel that observations $\{X_i\}$ form a stationary sequence obeying (4).

Note that conditions (3) and (4) are among the weakest in the literature on dependent random variables. In many particular parametric models, including GARCH process, $\varphi(\cdot)$ decays exponentially fast, see Basrak et al. (2002). Thus, papers assuming GARCH model implicitly require $\varphi(\cdot)$ to have the exponential rate of decay. Recall that data has "long memory" if $\varphi(k)$ decays not faster than k^{-d} for some $d > 0$. Hence GARCH model is not applicable to "long memory" data. The evidence of the long memory phenomenon for daily log–returns of S&P500 and some other financial data is provided by Ding et al. (1993).

Any quantity of interest in the tail appears a function of the tail index. Given the sample $X_1, ..., X_n$, we want to estimate the tail index, q–VaR (2), where $q = q(n)$ is allowed to tend to 0 as the sample size n grows, and the corresponding ES (denoted by $E(y_q)$).

All known estimators of VaR and ES in the tail seem to involve an estimator of the tail index. The problem of tail index estimation was addressed by many authors, see the survey paper by Novak (2005). Goldie & Smith (1987) have introduced the ratio estimator (RE)

$$a_n := a_n(x) = \sum_{i=1}^{n} \ln(X_i/x) 1\!\!1\{X_i > x\} \Big/ N_n(x) \tag{5}$$

of index $a = 1/\alpha$ (equivalently, $1/a_n$ is the ratio estimator of the tail index). Here n is the sample size, $x = x_n$ is the chosen threshold, and $N_n(x) = \sum_{i=1}^n 1\!\!1\{X_i > x\}$. Novak (2002, 2005) argues that RE has advantages over Hill's and some other tail index estimators. Resnick (1997), p.1839, and Embrechts et al. (1997), p. 406, point out drawbacks of Hill's and related estimators.

The statistic

$$a_{n,m} := a_{n,m}(x) = \sum_{i=1}^{n} \ln^m(X_i/x)1\!\!\mathrm{I}\{X_i > x\}\Big/(N_n(x)m!), \qquad (6)$$

where $m \in \mathbb{N}$, can be called a generalised ratio estimator. It is the sample analog of $\mathbb{E}\{\ln^m(X/x)|X > x\}/m!$. In fact, formula (6) generates a list of tail index estimators. For instance, $\tilde{a}_n = a_{n,2}/a_n$ is a consistent estimator of index a if (7) holds.

Theorem. *Denote $p_n = \mathbb{P}(X > x_n)$, and suppose that*

$$p_n \to 0\,, \ np_n \to \infty \qquad (7)$$

as $n \to \infty$. Then $a_{n,m}^{1/m}$ is a consistent estimator of index a.

Assumption (7) means that the threshold x_n is neither "too small" nor "too large". It guarantees the consistency of typical estimators implementing the so–called peak–over-threshold approach, see Novak (2005).

The ratio estimator is a function of the threshold. It is a typical situation in non–parametric statistics that an estimator is not initially a single number but a function of a nuisance parameter. The important practical question is how to choose the nuisance parameter and produce the final estimate as a number.

The procedure of practical estimation of index $a = 1/\alpha$: choose the interval $[x_-; x_+]$ formed by a significant number of sample points, where the function $a_{n,m}^{1/m}(\cdot)$ is stable, and take the average value $\hat{a} = \mathrm{mean}\{a_{n,m}^{1/m}(x) : x \in [x_-; x_+]\}$ of the generalised ratio estimator over that interval; then $\hat{x}_n$ is a level such that $a_{n,m}^{1/m}(\hat{x}_n) = \hat{a}$. Similarly we estimate VaR and ES.

The feature of this procedure is that it yields almost one and the same result despite possibly individual choice of the interval $[x_-; x_+]$: since we take the average over an interval formed by a significant number of sample points, the variability with the choice of end–points is virtually eliminated. Theorems establishing consistency of tail index, VaR and ES estimators (see Novak (2002, 2005)) form a theoretical background to this procedure.

We use in the next section the following estimators of VaR and ES (Novak (2002)):

$$\hat{y}_q(x_n) = (N_n/qn)^{a_n}\, x_n\,, \ E_n(x_n) = \hat{y}_q a_n/(1 - a_n)\,. \qquad (8)$$

The possible versions of estimators (8) are

$$\tilde{y}_q(x_n) = (N_n/qn)^{\hat{a}}\, x_n\,, \ y_q^* = \hat{y}_q(\hat{x}_n)\,, \ E_n^* = y_q^* \hat{a}/(1 - \hat{a})\,. \qquad (9)$$

Results of the simulation study in Novak (2002) indicate an acceptable degree of the accuracy of estimators $\hat{y}_q$, $\tilde{y}_q$ and y_q^*. The comparison of three different approaches in Novak (2005) and Novak & Beirlant (2006) suggests that (8), (9) are probably the best currently available estimator of VaR.

A number of authors suggested to use the bootstrap approach in order to choose the nuisance parameter when using Hill's estimator $a_n^H(k)$ of index a, see, e.g., Danielsson et al. (2001) and Matthys et al. (2004). One problem with the bootstrap is that it is designed for samples of *independent* random variables. Thus, the application of the bootstrap approach to the obviously dependent S&P500 data would not be justified. Another problem with the bootstrap is that it does not eliminate the nuisance parameter but replaces it with new ones, cf. Danielsson et al. (2001). The optimal value of the nuisance parameter k in Matthys et al. (2004) aims to minimize the asymptotic mean squared error (AMSE) of $\sqrt{k}\,(a_n^H(k)-a)$, while Drees (2003) minimizes the asymptotic variance only (ignoring the asymptotic bias). The problem is that the limiting distribution of $\sqrt{k}\,(a_n^H(k)-a)$ is determined by the rate of growth of $k=k(n)$, which we never know in practical situations. Moreover, for a particular rate of growth of k, the limiting distribution of $\sqrt{k}\,(a_n^H(k)-a)$ can be normal $\mathcal{N}(0;1)$ or can have infinite expectation; in both cases the optimal k cannot be determined by minimizing AMSE. For some other rates of k the limiting distribution of $\sqrt{k}\,(a_n^H(k)-a)$ is not normal at all.

4. September the 11$^{\text{th}}$

Forecasting the scale of possible extreme movements of stock prices and indexes is one of the major tasks of a risk manager. A particular question of this kind was raised in McNeil (1998): having the "historical" data available on the eve of the "black Monday", was it possible to predict the magnitude of the crash? The comparison of few approaches to this problem was given in Novak (2005) and Novak & Beirlant (2006); the procedure presented in Section 3 came up a clear winner.

We have analyzed daily and weekly log–returns of the S&P500 index over the period from 01.11.1987 till 10.09.2001 (3500 trading days, or 700 weeks). Figure 1 presents the plot of the ratio estimator for daily negative log–returns of the S&P500 index. The plot looks stable in the interval [1.6;4] which is formed by 163 points. The average value of $a_n(x)$ as $x\in[1.6;4]$ is $\hat{a}=0.294$. Hence the estimate of the tail index for daily data, α_d, is 3.4. We have estimated also 1%–VaR at 2.6% and the corresponding ES at 1.1%; the worst possible daily fall of the log–return of the S&P500 in 100 days was likely to be around VaR+ES=3.7%.

Comparing the tail index estimate with that for the period 01.01.1960 – 16.10.1987 (see Novak (2005), Novak and Beirlant (2006)), we conclude that *the left tail of daily log–returns of the S&P500 index became heavier*

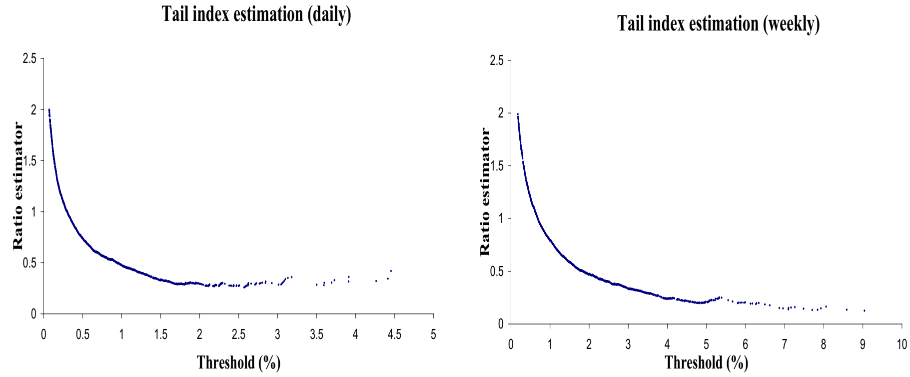

Figure 1: Tail index estimation for daily (left) and weekly (right) negative log–returns of S&P500 index over the period from 01.11.1987 till 10.09.2001. Daily data: the tail index α_d is estimated at 3.4. Weekly data: the tail index α_w is estimated at 4.5.

after the "black Monday" crash. Recall that the heavier the tail, the higher is the chance of an extreme movement.

In order to evaluate the worst possible *weekly* fall of the log–return of the S&P500 index after the "black Monday", we put $q = 1/700$. We have estimated the tail index, α_w, at 4.5 (see Figure 1), the q–VaR at 8.3% (see Figure 2) and the corresponding ES at 2.4%. Hence the worst possible fall of the weekly log–return of the S&P500 index in 700 weeks after the "black Monday" was likely to be around 8.3% + 2.4% = 10.7%.

The trade at New York Stock Exchange (NYSE) reopened on 17.09.2001. The S&P500 index finished the day at 1039, or 5% lower than its 1092.5 closing level on 10.09.2001. On 21.09.2001 the log–return of the index was 12.3% below its 10.09.2001 level. The fall during that extended week was larger than 10.7% predicted by our method. The possible explanation is that the market crash after September the 11th had two components: one (systematic) was determined by the historical data, while another (non–systematic) was due to the effect of the tragic event.

Recall that the tail index of the daily data, α_d, was estimated at 3.4 while the tail index of the weekly data, α_w, was estimated at 4.5. Thus, we have empirical evidence that financial data over different time horizons may have distinct tail indexes. This observation seems to be new, it has important implications to the problem of VaR estimation. It is known, see,

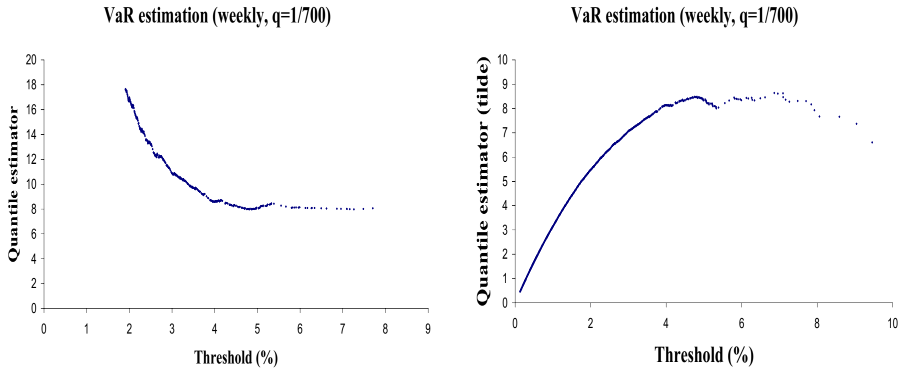

Figure 2: VaR estimation for weekly negative log–returns of S&P500 index over the period 01.11.1987 – 10.09.2001: $\hat{y}_q$ (left) and $\tilde{y}_q$ (right). The estimate of q–VaR is 8.3%, ES is estimated at 2.4%.

e.g., Embrechts et al. (1997), that for large enough x,

$$\bar{F}_k(x) := \mathbb{P}(X_1 + ... + X_k > x) \sim k\bar{F}(x) \tag{10}$$

(at least in the case of independent observations) if k is fixed and the distribution of X_i obeys (1*); in other words, the sum $X_1 + ... + X_k$ obeys (1*) *with the same tail index*.

Let X_i denote a daily log–return. Then a weekly log–return is a sum of five consecutive daily log–returns. Since Value–at–Risk over a k–days period, $\bar{F}_k^{-1}$, is the inverse to $\bar{F}_k$, (1*) and (10) yield

$$\bar{F}_k^{-1}(q) \sim k^{1/\alpha}\bar{F}^{-1}(q) \tag{11}$$

as $q \to 0$. Let q–VaR$_k$ denote q–VaR over a k–days period. Then (11) means

$$q\text{–VaR}_k \approx q\text{–VaR}_1 k^{1/\alpha}\,. \tag{12}$$

This simple relation, if it was true for real data, would mean that one could only need to evaluate, say, daily VaR and then use (12) in order to derive VaR over any desired time horizon. The latter is very attractive as sample sizes can get small when one deals with less frequent data.

In the case of normally distributed data, VaR$_k$ is a constant times σ_k, where σ_k is the standard deviation of a sum of k daily log–returns. Since $\sigma_k = \sigma_1\sqrt{k}$, many textbooks, e.g., Jorion (2001), Ho & Lee (2004), Tapiero (2004), recommend the formula

$$\text{VaR}_k \approx \text{VaR}_1\sqrt{k}\,. \tag{13}$$

Relation (12) means that one has to replace $\sqrt{k}$ in (13) with $k^{1/\alpha}$ if data is heavy–tailed.

The empirical evidence that the tail index of daily log–returns, α_d, is essentially different from the tail index of weekly log–returns, α_w, forces us to conclude that even formula (12) is not automatically applicable, and one should estimate VaR_k and ES_k separately for different values of k. Indeed, we have evaluated 1%–VaR_5 at 5.3%, while an application of (12) would give us $2.6\%{\cdot}5^{1/\alpha_d} = 4.2\%$.

Appendix

Proof of Theorem 1. Denote

$$a_m^* \equiv a_m^*(x) = \mathbb{E}\{\ln^m(X/x)|X > x\}/m!\,,\ \ \mathbb{I}_i \equiv \mathbb{I}_i(x) = \mathbb{I}\{X_i > x\}\,,$$

where $m \geq 1$, and let

$$Y_{i,m} = (\ln^m(X_i/x) - a_m^* m!)\,\mathbb{I}_i\,.$$

Property (1^*) yields $a_m^* \sim a^m$ as $x \to \infty$, cf. formula (7) in Novak (2002). Using Chebyshev's inequality, we derive

$$\begin{aligned}\mathbb{P}(a_{n,m} - a_m^* > \varepsilon) &= \mathbb{P}\left(\sum\nolimits_{i=1}^n Y_{i,m} > \varepsilon m! \sum\nolimits_{i=1}^n \mathbb{I}_i\right) \\ &= \mathbb{P}\left(\sum\nolimits_{i=1}^n \left(Y_{i,m} - \varepsilon m! \bar{\mathbb{I}}_i\right) > \varepsilon m! n p_n\right) \\ &\leq (\varepsilon m! n p_n)^{-2}\,\mathrm{var}\left(\sum\nolimits_{i=1}^n \left(Y_{i,m} - \varepsilon m! \bar{\mathbb{I}}_i\right)\right)\end{aligned}$$

for any $\varepsilon > 0$, where $\bar{\mathbb{I}}_i = \mathbb{I}_i - p_n$. By Theorem 1.1 in Utev (1989), there exists a constant c_ρ, depending only on $\rho(\cdot)$, such that

$$\mathrm{var} N_n \leq c_\rho n p_n\,,\ \ \mathrm{var}\left(\sum\nolimits_{i=1}^n Y_{i,m}\right) \leq c_\rho n \mathrm{var} Y_{1,m} \leq c_{\rho,m} n p_n\,.$$

Hence $\mathrm{var}\left(\sum_{i=1}^n \left(Y_{i,m} - \varepsilon m! \bar{\mathbb{I}}_i\right)\right) \leq C_{\rho,m} n p_n$, and $\mathbb{P}(a_{n,m} - a_m^* > \varepsilon) \to 0$ as $n \to \infty$. Similarly one checks that $\mathbb{P}(a_{n,m} - a_m^* < -\varepsilon) \to 0$ as $n \to \infty$. The result follows.

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