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ON SOME GENERALIZATIONS OF THE POLLACHEK–KHINCHINE FORMULA

For the skip-free Poisson process $\xi(t)$ $(t \geq 0, \xi(0) = 0)$,

$$\xi(t) = at + S(t),\ a < 0,\ S(t) = \sum_{k \leq \nu(t)} \xi_k,\ \xi_k > 0, \xi(0) = 0,$$

where $\nu(t)$ is a simple Poisson process with intensity $\lambda > 0$, the moment generating function (m.g.f.) of $\xi^+ = \sup_{0 \leq t < \infty} \xi(t)$ is defined by the well-known Pollachek–Khinchine formula under the condition $m = E\xi(1) < 0$ (see [1-3]).

For a homogeneous process $\xi(t)$ with bounded variation, we establish prelimit and limit generalizations of this formula, which define the m.g.f. of

$$\xi^+(\theta_s) = \sup_{0 \leq t \leq \theta_s} \xi(t),\ \xi^+ = \lim_{s \to 0} \xi^+(\theta_s) \left(P\{\theta_s > t\} = e^{-st},\ s > 0\right).$$

These generalizations are essentially based on the condition $P\{\tau^+(0) = \gamma^+(0) = 0\} = 0$, where $(\tau^+(0), \gamma^+(0))$ is the initial ladder point of $\xi(t)$ $(t \geq 0, \xi(0) = 0)$.

Some another relations for the m.g.f. of $\xi^+(\theta_s)$ and ξ^+ are established for the general lower semicontinuous process $\xi(t)$ on the base of results in [3-5].

1. Introduction

Let $\xi(t)$ be a lower continuous compound Poisson process with cumulant function $\psi(\alpha)$:

$$Ee^{i\alpha\xi(t)} = e^{t\psi(\alpha)},\ t \geq 0;\ \psi(\alpha) = i\alpha a + \lambda(\varphi(\alpha) - 1),$$

$$a < 0, \lambda > 0, \varphi(\alpha) = Ee^{i\alpha\xi_k},\ F(x) = P\{\xi_k < x\},\ x \geq 0, \overline{F}(x) = 1 - F(x).$$

Under the condition

$$m = E\xi(1) = a + \lambda\mu_1 < 0,\ \mu_1 = E\xi_k,\ F(0) = 0,$$

the moment generating function (m.g.f) of $\xi^+ = \sup_{0 \leq t < \infty} \xi(t)$ is defined by the classic Pollachek–Khinchine formula

$$Ee^{-z\xi^+} = \frac{p_+}{1 - q_+\widetilde{F}(z)/\mu_1},\ p_+ = P\{\xi^+ = 0\} = 1 - q_+,\ \widetilde{F}(z) = \int_0^\infty \overline{F}(x)e^{-zx}dx. \quad (1)$$

At first, we consider the process $\xi(t)$ with a bounded variation

$$\psi(\alpha) = i\alpha a + \int_{-\infty}^{\infty} (e^{i\alpha x} - 1)\Pi(dx),\ \int_{|x| \leq 1} |x|\Pi(dx) < \infty,\ a \leq 0, \quad (2)$$

and denote its extremal values and ladder points as

$$\xi^\pm(t) = \sup_{0 \leq t' \leq t}(\inf)\,\xi(t'),\ \xi^\pm = \sup_{0 \leq t < \infty}(\inf)\,\xi(t),\ \tau^-(-x) = \inf\{t > 0 : \xi(t) < -x\},$$

$$\tau^+(x) = \inf\{t > 0 : \xi(t) > x\},\ \gamma^+(x) = \xi(\tau^+(x)) - x,\ x \geq 0.$$

$$P\left\{\tau^+(x) < \infty\right\} = 1,\ \text{if } m \geq 0, P\left\{\tau^+(x) < \infty\right\} < 1,\ \text{if } m < 0.$$

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