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ON A STANDARD PRODUCT OF AN ARBITRARY FAMILY OF σ-FINITE BOREL MEASURES WITH DOMAINS IN POLISH SPACES

Let α be an infinite parameter set, and let $(\alpha_i)_{i\in I}$ be its any partition such that α_i is a non-empty finite subset for every $i \in I$. For $j \in \alpha$, let μ_j be a σ-finite Borel measure defined on a Polish metric space (E_j, ρ_j). We introduce a concept of a standard $(\alpha_i)_{i\in I}$-product of measures $(\mu_j)_{j\in\alpha}$ and investigate its some properties. As a consequence, we construct "a standard $(\alpha_i)_{i\in I}$-Lebesgue measure" on the Borel σ-algebra of subsets of $\mathbb{R}^\alpha$ for every infinite parameter set α which is invariant under a group generated by shifts. In addition, if card$(\alpha_i) = 1$ for every $i \in I$, then "a standard $(\alpha_i)_{i\in I}$-Lebesgue measure" m^α is invariant under a group generated by shifts and canonical permutations of $\mathbb{R}^\alpha$. As a simple consequence, we get that a "standard Lebesgue measure" $m^{\mathbb{N}}$ on $\mathbb{R}^{\mathbb{N}}$ improves R. Baker's measure [2].

Let $(X_i, \mathbf{B}_i, \mu_i)$ $(i \in \mathbb{N})$ be a family of regular Borel measure spaces, where X_i is a Hausdorff topological space. In [4], it was proved that a Borel measure μ exists on $\prod_{i\in\mathbb{N}} X_i$ (with respect to the product topology) such that if $K_i \subseteq X_i$ is compact for all $i \in \mathbb{N}$ and $\prod_{i\in\mathbb{N}} \mu_i(K_i)$ converges, then $\mu(\prod_{i\in\mathbb{N}} K_i) = \prod_{i\in\mathbb{N}} \mu_i(K_i)$. Note that a special case of this result (in the case where $X_i = R$ and m_i is Lebesgue measure) has been proved only recently in [1]. Slightly later on, work [2] has improved the result in [4] as follows: there exists of a Borel measure λ on $\prod_{i\in\mathbb{N}} X_i$ such that if $R_i \subseteq X_i$ is measurable for $i \in \mathbb{N}$ and $\prod_{i\in\mathbb{N}} \mu_i(R_i)$ converges, then $\lambda(\prod_{i\in\mathbb{N}} R_i) = \prod_{i\in\mathbb{N}} \mu_i(R_i)$.

Note that both above-mentioned constructions in the case where multiplied measures coincide with a specific σ-finite Borel measure μ in a Hausdorff topological space X give a measure $\mu^{\mathbb{N}}$ which is not invariant under permutations of the $X^{\mathbb{N}}$. To eliminate this defect, we introduce a notion of a *standard product of measures* and prove its existence under some assumptions. Our approach, unlike [4], [1],[2], is based on the notion of a standard product of a family of real numbers. Main results of the article are the theorem about the existence of a *standard product of measures* and its invariance under action of some group of transformations. In the case where multiplied measures coincide with a Lebesgue measure on $\mathbb{R}$, our product occurs to be invariant under permutations of the $\mathbb{R}^{\mathbb{N}}$ (see [7]) unlike Baker's measures [1],[2]. In addition, our construction is essentially different from the points of view of [4] and [2], because it allows one to construct a *standard product of measures* for an arbitrary (not only for countable) family of σ-finite Borel measures with domains in Polish spaces.

Suppose that X is a topological space. The Borel sets $\mathcal{B}(X)$ are the σ-algebra generated by the open sets of a topological space X, and the Baire sets $\mathcal{B}_0(X)$ are the smallest σ-algebra making all real-valued continuous functions measurable. In 1957 (see [5]), Mařík proved that all normal countably paracompact spaces have the following property: Every Baire measure extends to a regular Borel measure. Spaces which have this property have come to be known as Mařík spaces.

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