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BAXTER TYPE THEOREMS FOR GENERALIZED RANDOM GAUSSIAN PROCESSES

Some type of Baxter sums for generalized random processes are constructed in this work. Sufficient conditions for such a sum to converge to a non–random constant are obtained. We apply our result to a process of white noise and a derivative of fractional Brownian motion.

1. Introduction

Let $\xi(t), t \in [0,1]$ be a random process, $\lambda_n = \left\{0 = t_0, t_1, \ldots, t_{b(n)} = 1\right\}$ be a uniform partition of line segment $[0,1]$, $\Delta\xi(t_k) = \xi(t_{k+1}) - \xi(t_k), f : R \to [0,\infty)$ be a Borel function, $b(n) \in N, b(n) \to \infty, n \to \infty$. The sum

$$S\left(X, \lambda_n\right) = \sum_{k=0}^{b(n)-1} f\left(\Delta\xi(t_k)\right)$$

is called a Baxter sum. Further we consider only the case $f(x) = x^2, x \in R$.

Limit theorems dealing with one or another sense of convergence of properly normalized Baxter sums to non–random constants are called Baxter type theorems or theorems of Levy–Baxter type. Theorems of Baxter type for Gaussian random processes and fields were obtained by many authors. The pioneer works on this topic for Gaussian processes belong to P. Levy [1], G. Baxter [2], E. G. Gladyshev [3] and ones for Gaussian fields belong to S. M. Berman [4], S. M. Krasnitskiy [5], T. V. Arak [6]. Traditionally the Levy–Baxter type theorems refer to domain of stochastic analysis. Later these theorems were applied to some problems of statistics of random processes. For example the Baxter sum method was applied to the estimation of fractional Brownian motion Hurst parameter in works of O. O. Kurchenko [7], J–C Breton, I. Nourdin, G. Peccati [8]. This method was used for the estimation of covariation function parameters for multiparameter fractional Brownian fields in the article [9] by Yu. V. Kozachenko and O. O. Kurchenko. The part of monograph by B. L. S. Pracasa Rao [10] is devoted to the application of Baxter sums to statistics of random processes. The estimates obtained by the Baxter sum method have the property of consistency. One of the advantages of this method lies in the possibility to construct non-asymptotic confidence intervals. The Baxter sum method in statistics of generalized random functions was used as an example in monograph [11] by Yu. A. Rozanov, and also in articles [12] by V. B. Gorjainov, [13] by N. M. Arato. The theory of generalized random functions is rather completely exploited in the third chapter of monograph [14] by I. M. Gelfand and N. Vilenkin.

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2. Main results

Let K be the space of compactly supported infinitely differentiable functions on R, $\xi(\varphi) = (\xi, \varphi), \varphi \in K$ be a generalized Gaussian random process with zero mean. Subsequently we will use the two–parameter function families

$$\{\chi_{t,h} \in K : \chi_{t,h} = \chi_{t,h}(\cdot), t \in R, h > 0, supp\, \chi_{t,h} \subset [t, t+h]\}$$

and sequences $\{b(n)\} \subset N, b(n) \to \infty, n \to \infty$. Note that part of the overall results given in this paper (for example, Theorem 2.1 and 2.2), do not require further specification of the family of functions $\chi_{t,h}$. On the other hand, not every family of such functions is suitable for various classes of stochastic processes and different tasks. Some additional properties of functions $\chi_{t,h}$ (and sequences b_n) are shown below in the relevant parts of the text.

For the family $\{\chi_{t,h}\}$ and the sequence $\{b(n)\}$ we denote

$$\chi_{k,n} = \chi_{t,h}(\cdot)\big|_{t=k/b(n), h=1/b(n)}, k = 0, 1, \ldots, b(n)-1, n \geq 1.$$

We define

$$S_n(\xi) = \sum_{k=0}^{b(n)-1} (\xi, \chi_{k,n})^2, n \geq 1,$$

$$v_n(\xi) = \sum_{k,j=0}^{b(n)-1} (E(\xi, \chi_{k,n})(\xi, \chi_{j,n}))^2.$$

Theorem 2.1. *Let $\xi(\varphi)$ be a generalized Gaussian random process on K, $E\xi(\varphi) = 0, \varphi \in K$. Then the condition*

$$v_n(\xi) \to 0, n \to \infty$$

is necessary and sufficient to have

$$S_n(\xi) - ES_n(\xi) \to 0, \tag{1}$$

where the convergence takes place in the square mean. If

$$\sum_{n=1}^{\infty} v_n(\xi) < \infty,$$

then we have the almost sure convergence in (1).

Proof. Let us obtain the variance of Baxter sum $S_n(\xi)$:

$$VarS_n(\xi) = E(S_n(\xi) - ES_n(\xi))^2 = E(S_n(\xi))^2 - (ES_n(\xi))^2, n \geq 1.$$

We have the next equality:

$$E(S_n(\xi))^2 = \sum_{k,j=0}^{b(n)-1} E\left((\xi, \chi_{k,n})^2 (\xi, \chi_{j,n})^2\right).$$

For mathematical expectation of random values $\eta_1, \eta_2, \eta_3, \eta_4$ that have Gaussian joint distribution with zero mean value product we have [15, p. 29]:

$$E(\eta_1\eta_2\eta_3\eta_4) = E\eta_1\eta_2 E\eta_3\eta_4 + E\eta_1\eta_3 E\eta_2\eta_4 + E\eta_1\eta_4 E\eta_2\eta_3. \tag{2}$$

Substituting $\eta_1 = \eta_2 = (\xi, \chi_{k,n}), \eta_3 = \eta_4 = (\xi, \chi_{j,n})$ in the last equality, we get

$$E\left((\xi, \chi_{k,n})^2 (\xi, \chi_{j,n})^2\right) = 2(E(\xi, \chi_{k,n})(\xi, \chi_{j,n}))^2 + E(\xi, \chi_{k,n})^2 E(\xi, \chi_{j,n})^2,$$

$0 \le k, j \le b(n) - 1$. So we have

$$VarS_n(\xi) = 2 \sum_{k,j=0}^{b(n)-1} \left(E\left(\xi, \chi_{k,n}\right)\left(\xi, \chi_{j,n}\right)\right)^2 = 2v_n(\xi).$$

Thus,

$$VarS_n(\xi) = E\left(S_n(\xi) - ES_n(\xi)\right)^2 \to 0, n \to \infty$$

if and only if $v_n(\xi) \to 0$ as $n \to \infty$. Using this fact, we get the Theorem 2.1 statement about the convergence in the square mean. In the case of convergence of series

$$\sum_{n=1}^{\infty} v_n(\xi) = \frac{1}{2} \sum_{n=1}^{\infty} VarS_n(\xi)$$

the sequence $S_n(\xi) - ES_n(\xi)$ approaches to 0 with probability 1 when $n \to \infty$ ([16, p. 24]). The proof is over. □

Corollary 2.1. *If the conditions of the Theorem 2.1 are fulfilled and $ES_n(\xi) \to c \in R$ when $n \to \infty$, then the requirement $v_n(\xi) \to 0, n \to \infty$ is necessary and sufficient for the square mean convergence*

$$S_n(\xi) \to c, n \to \infty.$$

If $\sum_{n=1}^{\infty} v_n(\xi) < \infty$, then the convergence takes place almost surely.

Definition 2.1. The generalized random process is said to be a process with independent values if the random variables $(\xi, \varphi), (\xi, \psi)$ are independent for $\varphi, \psi \in K, \varphi(x)\psi(x) = 0$ for any $x \in \mathbb{R}$.

Corollary 2.2. *Let $\xi(\varphi)$ be the generalized Gaussian random process with independent values, $E\xi(\varphi) = 0, \varphi \in K$. Next, put*

$$v_n^{(0)}(\xi) = \sum_{k=0}^{b(n)-1} \left(E\left(\xi, \chi_{k,n}\right)^2\right)^2.$$

Then the condition

$$v_n^{(0)}(\xi) \to 0, n \to \infty$$

is necessary and sufficient for the convergence (1) in the square mean. If

$$\sum_{n=1}^{\infty} v_n^{(0)}(\xi) < +\infty,$$

then we have the almost sure convergence in (1).

Proof. Since $\chi_{k,n}(x)\chi_{j,n}(x) = 0$ for any $x \in \mathbb{R}, k \ne j$, we have $E\left(\xi, \chi_{k,n}\right)\left(\xi, \chi_{j,n}\right) = 0$ for $k \ne j$. For this reason, in this case $v_n(\xi) = v_n^{(0)}(\xi)$. The Corollary is proved. □

Example 2.1. Let $\xi = \xi(\varphi)$ be the process of white noise, i.e. a generalized Gaussian random process with zero mean and covariation function $E\xi(\varphi)\xi(\psi) = \int_{-\infty}^{\infty} \varphi(x)\psi(x)dx$. So ξ is the process with independent values. Further let a function $\chi_{t,h} : R \to [0,1]$ be defined for $t \in R, h > 0$ as

$$\chi_{t,h} \in K, supp\, \chi_{t,h} \subset (t, t+h), \chi_{t,h} = 1 \quad \text{for}\, x \in (t + h^2, t + h - h^2)$$

Then

$$E\left(\xi, \chi_{t,h}\right)^2 = \int_t^{t+h} \chi_{t,h}^2(x)dx = h + O(h^2), h \to 0.$$

For this reason

$$ES_n(\xi) = 1 + O\left(\frac{1}{b(n)}\right)$$

and

$$v_n^{(0)}(\xi) = \sum_{k=0}^{b(n)-1} \left(E\left(\xi, \chi_{k,n}\right)^2 \right)^2 = O\left(\frac{1}{b(n)} \right), n \to \infty.$$

On the account of Corollaries 2.1, 2.2 we have $S_n(\xi) \to 1$ in the square mean as $n \to \infty$. If the series

$$\sum_{n=1}^{\infty} \frac{1}{b(n)}$$

is convergent, then $S_n(\xi) \to 1$ almost surely.

Remark 2.1. Let $(\Omega, \sigma, P_1, P_2)$ be a statistical structure, i.e. Ω be an elementary events space, σ be a σ–algebra of events (subsets of Ω), P_1, P_2 be a probabilistic measures on (Ω, σ). Let $\sigma(\xi) \subset \sigma$ be a σ–algebra generated by the generalized random process $\xi = \xi(\varphi), \varphi \in K$. The process ξ is supposed to be Gaussian with respect to both measures P_1, P_2.

Definition 2.2. Further let E_1, E_2 be the symbols of mathematical expectation with respect to the measures P_1, P_2 respectively. Denote by $v_{i,n}(\xi), i = 1, 2$ the result of substitution of the symbol E_i into the $v_n(\xi)$ expression instead of E, $i = 1, 2$. Further, denote the restrictions of measures P_1, P_2 to the σ–algebra $\sigma(\xi)$ by $P_{1,\xi}, P_{2,\xi}$ respectively.

Corollary 2.3. *Let the process ξ and the measures P_1, P_2 satisfy the conditions of Remark 2.1, $E_1\xi(\varphi) = E_2\xi(\varphi) = 0$. Suppose that the next conditions (1) – (3) are fulfilled*

(1) $\sum_{n=1}^{\infty} v_{1,n}(\xi) < +\infty, \sum_{n=1}^{\infty} v_{2,n}(\xi) < +\infty;$
(2) $E_1 S_n(\xi) \to c_1, E_2 S_n(\xi) \to c_2, n \to \infty;$
(3) $c_1 \neq c_2$.

Then the measures $P_{1,\xi}, P_{2,\xi}$ are orthogonal (singular).

Proof. Let $X_i = \{\omega \in \Omega : S_n(\xi)(\omega) \to c_i, n \to \infty\}, i = 1, 2$. Since Theorem 2.1, it follows that $P_{1,\xi}(X_1) = 1, P_{2,\xi}(X_2) = 1$. But $X_1 \cap X_2 = \emptyset$. □

Theorem 2.2. *Suppose that the function family $\chi_{t,h} \subset K$ and the generalized Gaussian random process $\xi = \xi(\varphi)$ with zero mean satisfy the following conditions:*

(1) *For a sufficiently small positive h the function $E\left(\xi, \chi_{t,h}\right)^2$ is continuous for $t \in [0, 1]$ and there exist a Borel function $g : (0, +\infty) \to (0, +\infty), g(0+) = 0$ and a Borel function $u : [0, 1] \to [0, +\infty)$ such that*

$$\frac{E\left(\xi, \chi_{t,h}\right)^2}{g(h)} \to u(t), h \downarrow 0$$

uniformly over $t \in [0, 1]$;

(2) *For the sequences*

$$\alpha(n) \stackrel{\text{def}}{=} \frac{1}{b(n) g\left(\frac{1}{b(n)}\right)}, n \geq 1$$

and

$$v_n^{(1)}(\xi) \stackrel{\text{def}}{=} \alpha^2(n) \sum_{\substack{k,j=0, \\ |k-j| \geq 2}}^{b(n)-1} \left(E\left(\xi, \chi_{k,n}\right)\left(\xi, \chi_{j,n}\right) \right)^2$$

it is fulfilled that $v_n^{(1)}(\xi) \to 0, n \to \infty$.

Then

$$\tilde{S}_n(\xi) \stackrel{\text{def}}{=} \alpha(n) \sum_{k=0}^{b(n)-1} (\xi, \chi_{k,n})^2 \to \int_0^1 u(t)dt, n \to \infty \tag{3}$$

in the square mean. If the series

$$\sum_{n=1}^{\infty} v_n^{(1)}(\xi), \sum_{n=1}^{\infty} \frac{1}{b(n)} \tag{4}$$

are convergent, then the convergence in (3) takes place almost surely.

Proof. It follows from the condition (1) that

$$E\tilde{S}_n(\xi) \to \int_0^1 u(t)dt, n \to \infty. \tag{5}$$

Using the equality (2) as before in Theorem 2.1 proof, we get the expression for $\tilde{S}_n(\xi)$ variance:

$$Var\tilde{S}_n(\xi) = 2\alpha^2(n) \sum_{k,j=0}^{b(n)-1} \left(E\left(\xi, \chi_{k,n}\right)\left(\xi, \chi_{j,n}\right)\right)^2 =$$

$$= 2\alpha^2(n) \sum_{\substack{k,j=0,\\ |k-j|\le 1}}^{b(n)-1} \left(E\left(\xi, \chi_{k,n}\right)\left(\xi, \chi_{j,n}\right)\right)^2 +$$

$$+2\alpha^2(n) \sum_{\substack{k,j=0,\\ |k-j|\ge 2}}^{b(n)-1} \left(E\left(\xi, \chi_{k,n}\right)\left(\xi, \chi_{j,n}\right)\right)^2.$$

Due to the condition (2) the last summand tends to 0 with $n \to \infty$. Next,

$$\alpha^2(n) \sum_{\substack{k,j=0,\\ |k-j|\le 1}}^{b(n)-1} \left(E\left(\xi, \chi_{k,n}\right)\left(\xi, \chi_{j,n}\right)\right)^2 \le$$

$$\le \alpha^2(n) \sum_{\substack{k,j=0,\\ |k-j|\le 1}}^{b(n)-1} E\left(\xi, \chi_{k,n}\right)^2 E\left(\xi, \chi_{j,n}\right)^2 =$$

$$= \alpha^2(n) \sum_{\substack{k,j=0,\\ |k-j|\le 1}}^{b(n)-1} \left(u\left(\frac{k}{b(n)}\right) + o(1)\right)\left(u\left(\frac{j}{b(n)}\right) + o(1)\right) g^2\left(\frac{1}{b(n)}\right) = \Delta_n.$$

Here we used the Cauchy-Bunyakovskii inequality for estimation of $\left(E\left(\xi, \chi_{k,n}\right)\left(\xi, \chi_{j,n}\right)\right)^2$ and the condition (1). Now set $C = \sup_{t\in[0,1]} u(t) + 1$. The sum in the expression of Δ_n has $3b(n) - 2$ summands. Thus, for all sufficiently large n we have $\Delta_n \le \frac{3}{b(n)} C^2$. So $Var\tilde{S}_n(\xi) \to 0, n \to \infty$. Hence, it follows from (5) that (3) is fulfilled in the square mean. If the series (4) are convergent, then the series $\sum_{n=1}^{\infty} Var\tilde{S}_n(\xi)$ is convergent. Therefore, in this case, the convergence in (3) takes place with probability 1. The proof is over. □

Example 2.2. Let $\xi_H(t), t \in R$ be a Gaussian random process with zero mean and covariation function

$$B_H(s,t) = \frac{1}{2}\left(|s|^{2H} + |t|^{2H} - |s-t|^{2H}\right), s, t \in R, 0 < H < 1 \tag{6}$$

(the constant H is called a Hurst parameter). The process $\xi_H(t)$ is called the fractional Brownian motion with Hurst parameter H. We will consider the derivative $\eta_H = \xi_H'$ of this process as the generalized Gaussian random process on $K : (\eta_H, \varphi) = -(\xi_H, \varphi')$. The process η_H has zero mean and covariation function

$$B_H(\varphi, \psi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_H(s,t)\frac{d\varphi(s)}{ds}\frac{d\psi(t)}{dt}dsdt.$$

By $\mu_{t,h} = \mu_{t,h}(\cdot)$ denote the function from the space K with parameter $t \in R, h > 0$, that is determined, provided that h is sufficiently small, by the following conditions:

(1) $supp\,\mu_{t,h}(\cdot) \subset \left[t, t + \exp\left(-\frac{1}{h}\right)\right] \cup \left[t + h - \exp\left(-\frac{1}{h}\right), t + h\right]$;
(2) $0 \le \mu_{t,h}(s) \le \exp\left(\frac{1}{h}\right)$ on $\left[t, t + \exp\left(-\frac{1}{h}\right)\right]$;
(3) $\mu_{t,h}(s) = \exp\left(\frac{1}{h}\right)$ on $\left[t + \exp\left(-\frac{1}{h^2}\right), t + \exp\left(-\frac{1}{h}\right) - \exp\left(-\frac{1}{h^2}\right)\right]$;
(4) The graph of $\mu_{t,h}$ is centrally symmetric relative to the point $\left(t + \frac{h}{2}, 0\right)$.

Define the function family $\{\chi_{t,h}\}$ by the equality

$$\chi_{t,h}(x) = \int_{-\infty}^{x} \mu_{t,h}(s)ds, \quad x \in R.$$

As a result of the formula (6) and the stationarity of the derivative of fractional Brownian motion we get

$$E\left(\eta_H, \chi_{t,h}\right)^2 = E\left(\eta_H, \chi_{0,h}\right)^2 = E\left(\xi_H, \mu_{0,h}\right)^2 =$$
$$= \frac{1}{2}\int_0^h ds \int_0^h \left(s^{2H} + t^{2H} - |s-t|^{2H}\right)\mu_{0,h}(s)\mu_{0,h}(t)dt.$$

Directly integrating, we find that

$$E\left(\eta_H, \chi_{t,h}\right)^2 = h^{2H} + o(h^{2H}), h \to 0.$$

Thus, condition (1) of Theorem 2.2 holds for

$$u(t) = 1, t \in [0,1]; g(h) = h^{2H}.$$

Thus $\alpha(n) = (b(n))^{2H-1}$. Let us move on to the verification of condition (2). Due to the stationarity of the derivative of fractional Brownian motion we have

$$v_n^{(1)}(\eta_H) \stackrel{\text{def}}{=} \alpha^2(n) \sum_{\substack{k,j=0,\\ |k-j|\ge 2}}^{b(n)-1} \left(E\left(\eta_H, \chi_{k,n}\right)\left(\eta_H, \chi_{j,n}\right)\right)^2 =$$
$$= 2\alpha^2(n) \sum_{j=0}^{b(n)-3}\sum_{k=j+2}^{b(n)-1} \left(E\left(\eta_H, \chi_{k,n}\right)\left(\eta_H, \chi_{j,n}\right)\right)^2 =$$
$$= 2\alpha^2(n) \sum_{l=2}^{b(n)-1} \left(b(n) - l\right)\left(E\left(\eta_H, \chi_{0,n}\right)\left(\eta_H, \chi_{l,n}\right)\right)^2,$$

where $\chi_{k,n} = \chi_{kh,h}, h = \frac{1}{b(n)}$. Further,

$$E\left(\eta_H, \chi_{0,n}\right)\left(\eta_H, \chi_{l,n}\right) = E\left(\xi_H, \mu_{0,n}\right)\left(\xi_H, \mu_{l,n}\right) =$$
$$= \frac{1}{2}\int_0^h \mu_{0,n}(s)ds \int_{lh}^{(l+1)h} \left(t^{2H} + s^{2H} - (t-s)^{2H}\right)\mu_{l,n}(t)dt =$$
$$= -\frac{1}{2}\int_0^h \mu_{0,n}(s)ds \int_{lh}^{(l+1)h} (t-s)^{2H}\mu_{l,n}(t)dt =$$
$$= -\frac{1}{2}\int_0^h \tilde{\mu}_{0,n}(s)ds \int_{lh}^{(l+1)h} (t-s)^{2H}\tilde{\mu}_{l,n}(t)dt + O\left(\exp\left(-\frac{1}{h^2}\right)\exp\left(\frac{1}{h}\right)\right), h \to 0,$$

where $\tilde{\mu}_{k,n}(s) = \exp\left(\frac{1}{h}\right)\left(I_{\left[kh,kh+\exp\left(-\frac{1}{h}\right)\right]}(s) - I_{\left[(k+1)h-\exp\left(-\frac{1}{h}\right),(k+1)h\right]}(s)\right)$, $k = 0, 1, \ldots, b(n)-1$ and the O–large relation is fulfilled uniformly with respect to $l \in \{2, 3, \ldots, b(n)-1\}$. The direct integration permits to obtain the equality

$$-\frac{1}{2}\int_0^h \tilde{\mu}_{0,n}(s)ds \int_{lh}^{(l+1)h} (t-s)^{2H}\tilde{\mu}_{l,n}(t)dt = O\left(\frac{h^{2H}}{(l-1)^{2H}}\right), h \to 0$$

uniformly with respect to $l \in \{2, 3, \ldots, b(n)-1\}$.

Thus, for some $C > 0$ and for all sufficiently large n we have

$$v_n^{(1)}(\eta_H) \le C\,(b(n))^{4H-2}\,\frac{1}{(b(n))^{4H}}\,b(n)\sum_{l=2}^{b(n)-1}\frac{1}{(l-1)^{4H}} =$$

$$= \frac{C}{b(n)}\sum_{l=2}^{b(n)-1}\frac{1}{(l-1)^{4H}} \to 0, n \to \infty.$$

Since

$$\sum_{l=2}^{b(n)-1}\frac{1}{(l-1)^{4H}} = \begin{cases} O\left((b(n))^{-4H+1}\right), & 0 < H < \frac{1}{4}; \\ O(\ln b(n)), & H = \frac{1}{4}; \\ O(1), & \frac{1}{4} < H < 1, \end{cases}$$

as $n \to \infty$, then

$$v_n^{(1)}(\eta_H) = \begin{cases} O\left((b(n))^{-4H}\right), & 0 < H < \frac{1}{4}; \\ O\left(\frac{\ln b(n)}{b(n)}\right), & H = \frac{1}{4}; \\ O\left(\frac{1}{b(n)}\right), & \frac{1}{4} < H < 1. \end{cases}$$

That is, the condition (2) of Theorem 2.2 is satisfied. By the virtue of Theorem 2.2

$$(b(n))^{2H-1}\sum_{k=0}^{b(n)-1}(\eta_H, \chi_{k,n})^2 \to 1, n \to \infty \tag{7}$$

in the square mean. If for any $\varepsilon > 0$ the series

$$\sum_{n=1}^{\infty}\frac{1}{(b(n))^{\varepsilon}}$$

converges, the convergence in (7) occurs with probability 1.

For brevity, we will call the process $\xi_H(t)$ of Example 2.2 ξ_H–process.

Corollary 2.4. *Let the statistical structure $(\Omega, \sigma, P_1, P_2)$ be such that the process $\xi(t), t \in R$ is the ξ_{H_1}-process with respect to the measure P_1 and $\xi(t), t \in R$ is the ξ_{H_2}-process with respect to the measure P_2. Let $\eta = \eta(\varphi), \varphi \in K$ be the generalized random process that is the derivative of $\xi(t)$. In accordance to Definition 2.2 let $\sigma(\eta) \subset \sigma$ be the σ–algebra generated by the generalized process η, $P_{1,\eta}$ and $P_{2,\eta}$ be the restrictions of measures P_1, P_2 on the σ–algebra $\sigma(\eta)$ respectively. Then for $H_1 \neq H_2$ measures $P_{1,\eta}, P_{2,\eta}$ are orthogonal (singular).*

Proof. Corollary 2.4 follows from the limit relation (7) in the same way as Corollary 2.3 follows from Theorem 2.1. Indeed, let

$$X_i = \left\{\omega \in \Omega : (b(n))^{2H_i-1}\sum_{k=0}^{b(n)-1}(\eta, \chi_{k,n})^2 \to 1, n \to \infty\right\}, i = 1, 2,$$

where $b(n)$ tends to infinity sufficiently fast. We have the equalities $P_{1,\eta}(X_1) = P_{2,\eta}(X_2) = 1$. But $X_1 \bigcap X_2 = \emptyset$. □

It is known, that the condition $H_1 \neq H_2$ implies that $P_{1,\xi}, P_{2,\xi}$ are singular (see, for example, [5]). But this does not imply orthogonality of $P_{1,\eta}, P_{2,\eta}$ automatically, because the sigma-algebras $\sigma(\xi), \sigma(\eta)$, generally speaking, do not coincide with each other ([17]).

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