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THE EXPLICIT PROBABILITY DISTRIBUTION OF A SIX-DIMENSIONAL RANDOM FLIGHT

We consider the symmetric random motion with finite speed $\mathbf{X}(t)$ in the Euclidean space $\mathbb{R}^6$ subject to the control of a homogeneous Poisson process. The explicit probability distribution of $\mathbf{X}(t)$, $t > 0$, is obtained.

1. Introduction

The multidimensional diffusion with finite speed of propagation is generated by the finite-velocity random motions of a particle that moves in the Euclidean space $\mathbb{R}^m$, $m \geq 2$, and whose evolution is driven by some stochastic process. The most studied model is performed by the symmetric random motion controlled by a homogeneous Poisson process with the uniform choice of directions. Such a type of motion is referred to as the random flight or, in a more general sense, random evolution. One of the most important features of such a motion is that it generates an isotropic transport process in the Euclidean space $\mathbb{R}^m$ (see, for instance, Tolubinsky (1969), Papanicolaou (1975), Pinsky (1976)).

Random flights in the Euclidean spaces of different dimensions have thoroughly been examined in a series of works. In the study of such processes, the most desirable goal is undoubtedly their explicit distributions in the cases (very few, indeed) where such distributions can be obtained. The explicit form of the distribution of a two-dimensional symmetric random motion with finite speed was derived (by different methods) by Stadje (1987), Masoliver *et al.* (1993), Kolesnik and Orsingher (2005), and Kolesnik (2007). The distribution of a random flight in $\mathbb{R}^3$ was given by Tolubinsky (1969, Chapter 2, pp. 35-60) and by Stadje (1989) in rather complicated integral forms. Finally, the explicit form of the distribution of a random flight in $\mathbb{R}^4$ was obtained by Kolesnik (2006). The random flights in spaces of arbitrary higher dimensions were examined by Kolesnik (2008a); however, no new distributions were obtained in this work for higher dimensions $m \geq 5$.

Since the exact probability laws of random flights for lower dimensions were derived by rather complicated and sometimes tricky methods, the possibility of obtaining the explicit form of the distributions seemed very doubtful for higher dimensions $m \geq 5$.

However, a general unified method of studying the random flights in spaces of arbitrary dimensions was suggested in Kolesnik (2008a) based on the analysis of the integral transforms of their distributions. This method applied to the six-dimensional random motion enables us, surprisingly, to obtain the explicit probability law of the process, and this result is the core of the present paper. Although this method works for any dimension, the derivation of the *explicit* probability law in the space of such high dimension $m = 6$ looks like a "lucky accident" which, apparently, cannot be extended to higher dimensions.

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