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WEAK CONVERGENCE OF ADDITIVE FUNCTIONALS OF A SEQUENCE OF MARKOV CHAINS

We consider additive functionals $\phi_n, n \geq 1$ defined on a sequence of Markov chains that weakly converges to a Markov process. We give sufficient condition for $\phi_n, n \geq 1$ to converge in distribution, formulated in the terms of their characteristics (i.e. expectations). This condition generalizes Dynkin's theorem on convergence of W-functionals of a time homogeneous Markov process.

1. Introduction

Consider a sequence of processes $X_n = X_n(\cdot), n \geq 1$ that converge weakly, in an appropriate sense, to a process $X = X(\cdot)$. Consider also a family $\phi_n = \{\phi_n^{s,t}(X_n), s \leq t\}, n \geq 1$ of a functionals of the processes $X_n, n \geq 1$ and assume that the functionals are additive with respect to time variables. The general question, discussed in the present paper, is which information about the limit behavior of the distributions of functionals ϕ_n can be obtained in a situation where the processes X_n, X possess certain Markov properties. The starting point in our considerations is provided by the important particular case of the problem outlined above, in which all the processes X_n coincide with X. The well-known theorem by E.B. Dynkin ([1], Theorem 6.4) states that if X is a time homogeneous Markov process and ϕ_n are *W-functionals* of X (see [1], Chapter 6), then their limit behavior is completely determined by the limit behavior of their *characteristics* (that is, their expectations).

In the present paper, we consider the processes X_n that depend on n substantially. The class of a sequences $\{X_n\}$, considered in the framework of our approach, contains both sequences of Markov processes and sequences of Markov chains with appropriately re-scaled time, weakly convergent to Markov process X. An important partial case is provided by random broken lines corresponding to a random walk in $\mathbb{R}^d$ and weakly convergent to a time homogenous stable process X (particularly, to a Brownian motion).

We introduce a specific structural assumption on the sequence $\{X_n\}$ to provide *Markov approximation* for the process X. We show that, under this assumption, a full analogue of Dynkin's theorem holds: if the characteristics of a functional ϕ_n converge uniformly to the characteristic of a W-functional ϕ of the limit process X, then the distributions of ϕ_n converge weakly to the distribution of ϕ. Our method of proof is based on estimates for the L_2-distance between additive functionals, similar to those given in Lemma 6.5 [1]. These estimates are combined with the *coupling* technique, i.e. with a preliminary construction of processes X_n, X on one probability space in such a way that the functionals ϕ_n, ϕ, associated initially to different processes, are interpreted as functionals of a two-component process. The Markov property of the two-component process is essential for the estimates analogous to those given in Lemma 6.5 [1]; the assumption on the Markov approximation mentioned above is just the claim for such a property to hold true in an appropriate form.

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