EXPONENTIAL RATE OF L_p-CONVERGENCE OF INTRINSIC MARTINGALES IN SUPERCRITICAL BRANCHING RANDOM WALKS

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Abstract. Let $W_n, n \in \mathbb{N}_0$ be an intrinsic martingale with almost sure limit W in a supercritical branching random walk. We provide criteria for the L_p-convergence of the series $\sum_{n\geq 0} e^{an}(W - W_n)$ for $p > 1$ and $a > 0$. The result may be viewed as a statement about the exponential rate of convergence of $\mathbb{E}|W - W_n|^p$ to zero.

1. Introduction and main results

We start by recalling the definition of branching random walk. Consider a population starting from one ancestor located at the origin and evolving like a Galton–Watson process but with the generalization that individuals may have infinitely many children. All individuals are residing at points on the real line, and the displacements of children relative to their mother are described by a copy of a locally finite point process $\mathcal{M} = \sum_{i=1}^{J} \delta_{X_i}$ on $\mathbb{R}$, and, for different mothers, these copies are independent. Note once again that the random variable $J = \mathcal{M}(\mathbb{R})$ giving the offspring number may be infinite with positive probability. For $n \in \mathbb{N}_0 := \{0, 1, ...\}$, let $\mathcal{M}_n$ be the point process that defines the positions of the individuals of the n-th generation on $\mathbb{R}$. The sequence $\mathcal{M}_n, n \in \mathbb{N}_0$ is called a *branching random walk (BRW)*. In what follows, we always assume that $\mathbb{E}J > 1$ (supercriticality) which ensures the survival of the population with positive probability.

Every BRW is uniquely associated with a *weighted branching process (WBP)* to be formally introduced next: Let $\mathbf{V} := \bigcup_{n\geq 0} \mathbb{N}^n$ be the infinite Ulam–Harris tree of all finite sequences $v = v_1...v_n$ with root $\varnothing$ ($\mathbb{N}^0 := \{\varnothing\}$) and edges connecting each $v \in \mathbf{V}$ with its successors vi, $i = 1, 2, ...$ The length of v is denoted as $|v|$. Call v an individual and $|v|$ its generation number. Associate a nonnegative random variable $L_i(v)$ (weight) with every edge (v, vi) of $\mathbf{V}$ and define recursively $L_\varnothing := 1$ and $L_{vi} := L_i(v)L_v$. The random variable L_v can be interpreted as the total multiplicative weight assigned to the unique path from the root $\varnothing$ to v. For any $u \in \mathbf{V}$, put similarly $L_\varnothing(u) := 1$ and $L_{vi}(u) := L_i(v)L_v(u)$. Then $L_v(u)$ gives the total weight of the path from u to uv. Provided that $L_i(v), v \in \mathbf{V}, i \in \mathbb{N}$, consists of i.i.d. random variables, the pair $(\mathbf{V}, \mathbf{L})$ with $\mathbf{L} := (L_v(w)), v \in \mathbf{V}, w \in \mathbf{V}$ is called a WBP with associated BRW $\mathcal{M}_n, n \in \mathbb{N}_0$ defined as $\mathcal{M}_n = \sum_{|v|=n} \delta_{\log L_v}(\cdot \cap \mathbb{R})$. The quantities $\log L_v > -\infty$ for $v \in \mathbb{N}^n$ are thus the positions of the individuals alive in the generation n. Note that, if $u\mathbf{V} := \{uv : v \in \mathbf{V}\}$ denotes the subtree of $\mathbf{V}$ rooted at u, then the WBP on this subtree is given by $(u\mathbf{V}, \mathbf{L}(u))$, where $\mathbf{L}(u) := (L_u(v)), v \in \mathbf{V}$.

Next, we define

$$Z_n := \sum_{|v|=n} L_v \quad \text{and} \quad m(r) := \mathbb{E} \sum_{|v|=n} L_v^r$$

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