Theory of Stochastic Processes
Vol.14 (30), no.3-4, 2008, pp.67-76

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STORAGE PROCESSES IN POISSON APPROXIMATION SCHEME

Discrete storage processes, given by a sum of random variables on Markov and semi-Markov processes, are approximated by the Poisson compound processes on increasing time intervals.

Introduction

Renewal storage process (RSP) defined by a sum of independent identically distributed random variables $\alpha_n, \ n \geq 1$ taking values in Euclidean space R^d

$$\rho(t) = u + \sum_{n=1}^{\nu(t)} \alpha_n, \qquad t \geq 0,$$

where the counting renewal process $\nu(t) = \max\{n : \ \tau_n \leq t \}, \ t \geq 0,$ is defined by renewal moments $\tau_n \ \ n \geq 0, \ (\tau_0 = 0)$ on real line $R_+ = [\ 0, \ +\infty \ )$.

RSP has various interpretations in applications [1-3]. The main problem is to investigate the behavior of the RSP on increasing time intervals as $t \to \infty$. An effective method is to introduce the parameter series $\varepsilon \to 0 \ \ (\varepsilon > 0)$ in such a way that the limit theorems for stochastic processes may be used [1-5].

Asymptotic analysis of random evolution process is the most effective approach to get limit result for RSP in the series scheme. The theorem of Poisson approximation for RSP is realized under different assumptions for the renewal process $\nu(t), \ t \geq 0,$ driven by Markov or semi-Markov processes.

2000 *Mathematics Subject Classifications.* 60G50.
Key words and phrases. Discrete storage processes, sum of random variables, Markov and semi-Markov processes, approximation, Poisson compound process.

1.1. Renewal processes with Poisson jumps

Storage processes (SP) in the series scheme with small parameter series $\varepsilon \to 0 \quad (\varepsilon > 0)$ are given by relation

$$\rho^\varepsilon(t) = u + \sum_{n=1}^{\nu(t/\varepsilon)} \alpha_n^\varepsilon \ , \quad t > 0 \tag{1.1}$$

where the counting process

$$\nu(t) = \max\{n : \ \tau_n \le t\}, \qquad \tau_n = \textstyle\sum_{k=1}^n \theta_k, \quad n \ge 0, \quad \tau_0 = 0,$$

defined by i.i.d. random variables $\theta_k, \ \ k \ge 0$ with the distribution function $G(t) = P(\theta_k \le t)$, $G(0) = 0$. The random variables $\alpha_n^\varepsilon, \ \ n \ge 1$, take values at the real line (or in $R^d, d > 1$). The Poisson approximation conditions (PAC) (see [1, Ch. 7]) are given for the distribution functions $\Phi^\varepsilon(u) = P\{\alpha_n^\varepsilon < u\}, \quad u \in R$.

PAC 1: Approximation of distribution functions:

$$\int_R g(u)\Phi^\varepsilon(du) = \varepsilon\left[\Phi_g + \theta_g^\varepsilon\right], \quad g(u) \in C_3(R)$$

$C_3(R)$ is the measure determining class:

$$\Phi_g = \int_R g(u)\Phi(du).$$

PAC 2: Approximation of mean values:

$$\int_R u\Phi^\varepsilon(du) = \varepsilon\left[a + \theta_a^\varepsilon\right], \qquad \textstyle\int_R u^2\Phi^\varepsilon(du) = \varepsilon\left[c + \theta_c^\varepsilon\right].$$

The negligible terms $|\theta_\bullet^\varepsilon| \to 0$ when $\varepsilon \to 0$.

Theorem 1. *Under the conditions PAC 1-2 the weak convergence*

$$\rho^\varepsilon(t) \Rightarrow \rho^0(t), \quad \varepsilon \to 0$$

takes place. The limit compound Poisson process

$$\rho^0(t) = u + bt + \sum_{n=1}^{\nu^0(t)} \alpha_n^0 \ , \quad t \ge 0. \tag{1.2}$$

The distribution functions $\Phi_0(u) = P(\alpha_n^0 < u)$ of i.i.d. random variables $\alpha_n^0, \ \ n \ge 1,$ defined by the relation

$$\mathrm{E}g\left(\alpha_k^0\right) = \int_R g(u)\Phi^0(du) = \Phi_g/\Phi(R). \tag{1.3}$$

The counting Poisson process $\nu^0(t)$, $t \geq 0$, *is given by the intensity*

$$\mathrm{E}\nu^0(t) = q_0 t, \quad q_0 = q\Phi\left(R\right) = q\Lambda, \quad q = 1/\mathrm{E}\theta, \qquad \Lambda := \Phi\left(R\right) \tag{1.4}$$

The parameter of counting drift

$$b = q\left(a - \Lambda a^0\right), \qquad a^0 = \mathrm{E}\alpha_k^0. \tag{1.5}$$

Remark 1. The limit compound Poisson process (1.2) can be represented as follows

$$\rho^0(t) = u + qat + \sum_{n=1}^{\nu^0(t)} \tilde{\alpha}_n, \qquad \tilde{\alpha}_n = \alpha_n^0 - a^0. \tag{1.6}$$

Example 1. $\Phi^\varepsilon(au) = \begin{cases} \varepsilon\Lambda, & du = a^0, \\ 1-\varepsilon\Lambda & du = \varepsilon a_0, \end{cases}$ $\mathrm{E}\alpha_n^\varepsilon = \varepsilon\left(\Lambda a^0 + a_0\right) + \varepsilon\theta_a^\varepsilon,$ $a = a_0 + \Lambda a^0$, $\Phi_g = \Lambda g\left(a^0\right)$, $\Phi^0(g) = \Phi(g)/\Lambda$, $\alpha_n^0 = a^0$, $b = q(a-$ $-\Lambda \mathrm{E}\alpha_n^0) = q\left(a - \Lambda a^0\right) = qa_0$.

Remark 2. The intensity $q_0 = q\Lambda$ is proportional to average intensity of the renewal moments and the intensity Λ of big jumps of the sum (1.1).

Remark 3. Under the conditions PAC 1-2 the small jumps (εa_0) are transformed into continuous drift, and the big jumps (a^0) are get as jumps of the limit compound Poisson process.

1.2. Predictable characteristics of storage process

It is easy to calculate the predictable characteristics of the storage process (1.1) [4]: $B^\varepsilon(t) = \varepsilon\nu\left(t/\varepsilon\right)\left[a + \theta_b^\varepsilon\right]$, $C^\varepsilon(t) = \varepsilon\nu\left(t/\varepsilon\right)\left[c + \theta_c^\varepsilon\right]$, $\Phi_g^\varepsilon(t) = \varepsilon\nu\left(t/\varepsilon\right)\left[\Phi_g + \theta_g^\varepsilon\right]$. According to renewal theorem [7, Ch. 9]: $\varepsilon\nu\left(t/\varepsilon\right) \Rightarrow qt$, $\varepsilon \to 0$, $q = 1/\mathrm{E}\theta$.

Under the conditions of Theorem 1 we have the following limit results for $\varepsilon \to 0$: $B^\varepsilon(t) \Rightarrow qat$, $C^\varepsilon(t) \Rightarrow qct$, $\Phi_g^\varepsilon(t) \Rightarrow q\Phi_g t = q\Lambda\Phi_g^0 t$. $\Phi_g^\varepsilon(t) \Rightarrow q\Phi_g t = q\int_R g(u)\Phi\left(du\right)t = q\Lambda\int_R g(u)\Phi^0(du)t$. Here $\Phi^0(du) = \Phi\left(du\right)/\Phi(R)$; $\Lambda := \Phi(R)$.

Now the predictable characteristics $B^0(t) = qat$, $C^0(t) = qct$, $\Phi_g^0(t) = q\Lambda\Phi_g^0 t$ define the limit compound Poisson process with the drift:

$$\rho^0(t) = u + aqt + \sum_{n=1}^{\nu^0(t)} \left(\alpha_n^0 - a^0\right), \qquad t \geq 0$$

or, another form, is (1.2) with

$$b = q\left(a - \Lambda a^0\right), \quad a^0 = \mathrm{E}\alpha_n^0 = \int_R u\Phi^0(du)$$

2.1. Storage process at Markov process

Markov storage process (MSP) in a series scheme is defined as follows

$$\rho^\varepsilon(t) = u + \sum_{n=1}^{\nu(t/\varepsilon)} \alpha_n^\varepsilon\left(\kappa_n\right), \qquad t \geq 0 \tag{2.1}$$

where Markov process $\kappa\left(t\right), \ \ t \geq 0$, at a standard phase space (E, ε) is given by a generator [1, Ch. 1]

$$Q\varphi(x) = q(x)\int_E P\left(x, dy\right)\left[\varphi(y) - \varphi(x)\right]. \tag{2.2}$$

Counting process

$$\nu(t) := \max\left\{n : \tau_n \leq t\right\}, \qquad \tau_{n+1} = \tau_n + \theta_{n+1}, \quad n \geq 0 \tag{2.3}$$

and renewal moments θ_n are defined by conditional distribution functions

$$G_x(t) := P\left(\theta_x \leq t\right) = P\left\{\theta_{n+1} \leq t\left|\kappa_n = x\right.\right\} =$$
$$= 1 - e^{-q(x)t}, \ t \geq 0, \ x \in E. \tag{2.4}$$

The embedded Markov chain (EMC) $\kappa_n, \quad n \geq 0$ is defined by a stochastic kernel

$$P\left(x, B\right) = P\left(\kappa_{n+1} \in B\left|\kappa_n = x\right.\right) \ , \quad x \in E, \quad B \in \varepsilon. \tag{2.5}$$

We suppose that the EMC is uniformly ergodic with the stationary distribution $\rho\left(B\right), \quad B \in \varepsilon$. The family of random variables $\alpha_n^\varepsilon(x), x \in E, n \geq 1$ is defined by a family of distribution functions

$$\Phi_x^\varepsilon(u) = P\left(\alpha_n^\varepsilon(x) < u\right), \qquad u \in R, \quad x \in E. \tag{2.6}$$

The conditions of Poisson approximation are also supposed [1, Ch. 7]:

PAC1: $\int_R g(u)\Phi_x^\varepsilon(du) = \varepsilon\left[\Phi_g(x) + \theta_g^\varepsilon(x)\right], \quad g(u) \in C_3\left(R\right),$

PAC2: $\int_R u\Phi_x^\varepsilon(du) = \varepsilon\left[a(x) + \theta_a^\varepsilon(x)\right], \ \int_R u^2\Phi_x^\varepsilon(du) = \varepsilon\left[c(x) + \theta_c^\varepsilon(x)\right]$

with the negligible terms $\sup_{x \in E}|\theta_\bullet^\varepsilon(x)| \to 0$, $\varepsilon \to 0$.

Theorem 2. *Under the conditions PAC 1-2 the storage process (2.1) converges weakly to a compound Poisson process*

$$\rho^0(t) = u + bt + \sum_{n=1}^{\nu^0(t)} \alpha_n^0, \qquad t \geq 0. \tag{2.7}$$

Distribution function $\Phi^0(u) = \Phi(u)/\Phi(R) = P(\alpha_n^0 < u)$ *of i.i.d. random variables* $\alpha_n^0, \ n \geq 1,$ *is defined as*

$$\Phi(u) = q \int_E \rho(dx)\Phi_x(u), \quad \Phi_g(x) = \int_R g(u)\Phi_x(du), \qquad g \in C_3(R).$$

The compound Poisson process $\nu^0(t)$, $t \geq 0$ *is given by the intensity*

$$\mathrm{E}\nu^0(t) = q_0 t, \ q_0 = q\Lambda, \ \Lambda := \Phi(R).$$

The velocity of continuous drift $b = q(a - \Lambda a^0), \quad a^0 = \mathrm{E}\alpha_n^0$ *. The average intensity of Markov process*

$$q = \int_E \pi(dx)q(x), \qquad \pi(dx)\, q(x) = q\rho(dx),$$

where $\pi(B)$, $B \in \varepsilon$ *is the stationary distribution of Markov process* $\kappa(t)$, $t \geq 0$.

2.2. Predictable characteristics of Markov storage process (MSP)

According to the theorem about the representation of semimartingale (see [4, Ch. 2]), predictable characteristics of MSP are given as:

$$B^\varepsilon(t) = \sum_{n=1}^{\nu[t/\varepsilon]} \mathrm{E}\left[\alpha_n(\kappa_n)\,|F_{n-1}\right],$$

$$C^\varepsilon(t) = \sum_{n=1}^{\nu[t/\varepsilon]} \mathrm{E}\left[\alpha_n^2(\kappa_n)\,|F_{n-1}.\right], \tag{2.8}$$

$$\Phi_g^\varepsilon(t) = \sum_{n=1}^{\nu[t/\varepsilon]} \mathrm{E}\left[g\left(\alpha_n^\varepsilon(\kappa_n)\right)|F_{n-1}\right],$$

where $F_{n-1} := \sigma\left\{\ \kappa_r, \quad r \leq n-1\ \right\}, \quad n \geq 1$ is a family of σ-algebras.

According to the main assumptions PAC 1-2, the predictable characteristics of MSP have the following form

$$B^\varepsilon(t) = B_0^\varepsilon(t) + \theta_b^\varepsilon(t), \ C^\varepsilon(t) = C_0^\varepsilon(t) + \theta_0^\varepsilon(t), \ \Phi_g^\varepsilon(t) = \Phi_{g,0}^\varepsilon(t) + \theta_g^\varepsilon(t), \tag{2.9}$$

where the main parts are normalized increment processes

$$B_0^\varepsilon(t) = \varepsilon \sum_{n=1}^{\nu(t/\varepsilon)} a(\kappa_n), \ C_0^\varepsilon = \varepsilon \sum_{n=1}^{\nu(t/\varepsilon)} c(\kappa_n), \ \Phi_{g,0}^\varepsilon(t) = \varepsilon \sum_{n=1}^{\nu(t/\varepsilon)} \Phi_g^\varepsilon(\kappa_n).$$

Now the weak convergence of predictable characteristics (2.8) is equivalent to the weak convergence of normalized processes with increments (2.10) that follows from the Theorem 3.2 [1] . Limit predictable characteristics are the following:

$$B^0(t) = a^0 t, \quad C^0(t) = c^0 t, \quad \Phi_g^0(t) = \Phi_g^0 t, \tag{2.11}$$

where

$$a^0 = qa, \quad c^0 = qc, \quad \Phi_g^0 = q\Phi_g, \quad \Phi_g = \Phi_g^0 \Lambda, \tag{2.12}$$

$$a = \int_E \rho(dx)a(x), \quad c = \int_E \rho(dx)c(x), \quad \Phi_g = \int_E \rho(dx)\Phi_g(x). \tag{2.13}$$

Predictable characteristics (2.11)-(2.13) define the limit compound Poisson process (2.7).

3.1. Semi-Markov storage process (SMSP)

SMSP in a series scheme is defined by a correlation (as in (2.1))

$$\rho^\varepsilon(t) = u + \sum_{n=1}^{\nu(t/\varepsilon)} \alpha_n^\varepsilon(\kappa_n) \quad , \quad t \geq 0 \tag{3.1}$$

with semi-Markov switching process $\kappa(t), \quad t \geq 0,$ that is defined by a semi-Markov kernel [1, Ch. 1]

$$Q(x, B, t) = P(x, B)\, F_x(t) \quad , \quad x \in E, \quad B \in \varepsilon, \quad t \geq 0 \tag{3.2}$$

Stochastic kernel $P(x, B), \quad x \in E, \quad B \in \varepsilon$ defines the transition probabilities of embedded Markov chain $\kappa_n, \quad n \geq 0.$

Counting process

$$\nu(t) = \max\left\{ \, n : \quad \tau_n \leq t \, \right\}, \quad n \geq 0 \tag{3.3}$$

is defined by renewal moments

$$\tau_{n+1} = \tau_n + \theta_{n+1}, \quad n \geq 0$$

where the times between renewing $\theta_{n+1}, \quad n \geq 0$ are defined by conditional distribution functions

$$F_x(t) = P\left(\theta_{n+1} \leq t \,|\kappa_n = x\right) =: P\left(\theta_x \leq t.\right) \tag{3.4}$$

The main assumption is that SMP $\kappa(t), \quad t \geq 0$ is uniformly ergodic with stationary distribution $\pi(B), \quad B \in \varepsilon,$ that satisfies the correlation

$$\pi(dx)q(x) = q\rho(dx), \quad q = \int_E \pi(dx)q(x), \tag{3.5}$$

where the averaged intensity

$$q(x) = 1/m(x), \quad m(x) = \int_0^\infty \bar{F}_x(t)dt, \quad \bar{F}_x(t) := 1 - F_x(t). \qquad (3.6)$$

Stationary distribution $\rho\,(dx)$ of EMC $\kappa_n, \ \ n \geq 0$ satisfies the correlation

$$\rho(B) = \int_E \rho(dx)P\,(x, B)\,, \quad B \in \varepsilon, \quad \rho\,(E) = 1.$$

The family of random variables $\alpha_n^\varepsilon(x), \ \ x \in E, \ \ n \geq 1$ that are independent in general, is defined by the distribution function $\Phi_x^\varepsilon(du) = P\,(\alpha_n^\varepsilon(x) \in du)$.

Theorem 3. *The conditions of Poisson approximation are the following:*

$$\text{PAC 1:} \int_R u\Phi_x^\varepsilon(du) = [a(x) + \theta_a^\varepsilon(x)]\,, \qquad \int_R u^2\Phi_x^\varepsilon(du) = \varepsilon\,[c(x) + \theta_c^\varepsilon(x)]\,,$$

$$\text{PAC 2:} \ \int_R g(u)\Phi_x^\varepsilon(du) = \varepsilon\left[\Phi_g(x) + \theta_g^\varepsilon(x)\right], \quad g(u) \in C_3\,(R)\,,$$

$$\Phi_g(x) = \int_R g(u)\Phi_x(du).$$

Under the conditions PAC 1-2 the following weak convergence

$$\rho^\varepsilon(t) \Rightarrow \rho^0(t), \quad \varepsilon \to 0$$

takes place.

The limit compound Poisson process $\rho^0(t)$ is defined by its predictable characteristics

$$B^0(t) = b^0 t, \quad C^0(t) = c^0 t, \quad \Phi_g^0(t) = q\Phi_g t \qquad (3.7)$$

where

$$\Phi_g = \int_E \rho(dx)\Phi_g(x) = \Lambda\Phi_g^0\ , \quad b^0 = qb, \quad c^0 = qc$$

$$b = \int_E \rho(dx)a(x)c = \int_E \rho(dx)c(x), \Lambda = \Phi_g(R), \ \ \Phi_g = \int_R g(u)\Phi_g(du). \quad (3.8)$$

3.2. Predictable characteristics of SMSP

Predictable characteristics of SMSP (3.1) have the following form:

$$B^\varepsilon(t) = \sum_{n=1}^{\nu(t/\varepsilon)} \mathrm{E}\left[\alpha_n^\varepsilon\,(\kappa_n)\,\middle|\mathrm{F}_{n-1}\right]$$

$$C^\varepsilon(t) = \sum_{n=1}^{\nu(t/\varepsilon)} \mathrm{E}\left[(\alpha_n^\varepsilon(\kappa_n))^2 \left| \mathrm{F}_{n-1}\right.\right] \tag{3.9}$$

$$\Phi_g^\varepsilon(t) = \sum_{n=1}^{\nu(t/\varepsilon)} \mathrm{E}\left[g(\alpha_n^\varepsilon(\kappa_n)) \left| \mathrm{F}_{n-1}\right.\right].$$

According to the assumptions PAC 1-2 predictable characteristics (3.9) are the following

$$B^\varepsilon(t) = B_0^\varepsilon(t) + \theta_b^\varepsilon(t), C^\varepsilon(t) = C_0^\varepsilon(t) + \theta_c^\varepsilon(t), \Phi_g^\varepsilon(t) = \Phi_{g,0}^\varepsilon(t) + \theta_g^\varepsilon(t), \tag{3.10}$$

where

$$B_0^\varepsilon(t) = \varepsilon \sum_{n=1}^{\nu(t/\varepsilon)} a(\kappa_n), C_0^\varepsilon(t) = \varepsilon \sum_{n=1}^{\nu(t/\varepsilon)} c(\kappa_n) \Phi_{g,0}^\varepsilon(t) = \varepsilon \sum_{n=1}^{\nu(t/\varepsilon)} \Phi_g(\kappa_n) \tag{3.11}$$

and negligible terms $|\theta_\bullet^\varepsilon(t)| \to 0$ when $\varepsilon \to 0$.

Now the process of increments (3.11) at Markov chain $\kappa_n, \quad n \geq 0$ converges weakly at $\varepsilon \to 0$ according to Theorem 3.2. [1, Ch. 1]

$$B_0^\varepsilon(t) \Rightarrow \hat{a}t, \quad C_0^\varepsilon(t) \Rightarrow \hat{c}t, \quad \Phi_0^\varepsilon(t) \Rightarrow \Phi_g t \tag{3.12}$$

.

Under the conditions PAC 1-2 and main assumptions the following weak convergence of predictable characteristics takes place:

$$B^\varepsilon(t) \Rightarrow b^0(t), \quad C^\varepsilon(t) \Rightarrow c^0 t, \quad \Phi_g^\varepsilon(t) \Rightarrow \Phi_g t$$

where $b^0, \quad c^0$ and Φ_g are defined in (3.7)-(3.8).

The limit predictable characteristics define the limit compound Poisson process $\rho^0(t)$ in Theorem 3 with predictable characteristics (3.7).

4. Storage processes superposition of two renewal processes.

4.1. The superposition of two renewal processes is given by two sequences of sums (see [2, Ch. 1])

$$\tau_n^{(i)} = \textstyle\sum_{k=1}^n \theta_k^{(i)}, \quad n \geq 1, \quad \tau_0^{(i)} = 0, \quad i = 1, 2 \tag{4.1}$$

of i.i.d. positive random variables $\theta_k^{(i)}, \quad k \geq 1, \quad i = 1, 2$, defined by distribution functions $P_i(t) = P\left\{\theta_k^{(i)} \leq t\right\}, \quad P_i(0) = 0, \quad i = 1, 2$.

The superposition of two renewal processes is defined by a sum

$$\nu(t) = \nu_1(t) + \nu_2(t) \tag{4.2}$$

where $\nu_i(t) = \max\left\{ n : \ \tau_n^{(i)} \le t \right\}, \quad i = 1, 2$.

The superposition of two renewal processes (4.2) may be described using a semi-Markov process

$$\kappa(t), \quad t \ge 0$$

at a phase space

$$E = \left\{ ix, \quad i = 1, 2, \quad x > 0 \right\}, \quad \theta_{ix} = \theta^{(i)} \wedge x.$$

The first integer component i stands for an index of renewal moment, the second continuous component $x > 0$ stands for the time left till the moment of renewing with another index. The embedded Markov process $\kappa_n = \kappa\left(\tau_n\right), \quad n \ge 0$, is defined by a transition probability matrix (see [2, Par. 1.2.4])

$$P = \left[\begin{array}{cc} P_1\left(x - dy\right) & P_1\left(x + dy\right) \\ P_2\left(x + dy\right) & P_2\left(x - dy\right) \end{array} \right]. \tag{4.3}$$

The distinguishing specialty of embedded Markov chain $\kappa_n, \quad n \ge 0$, with transition probabilities (4.3) is its ergodicity with the stationary distribution

$$\rho_1(dx) = \rho_1 P_2^*(x)dx, \quad \rho_2(dx) = \rho_2 P_1^*(x)dx \tag{4.4}$$

where by the definition

$$P_i^*(x) := \bar{P}_i(x)/m_i, \quad \bar{P}_i(x) := 1 - P_i(x) \ ,$$

$$\rho_1 = \rho m_2, \ \rho_2 = \rho,$$

here $m_i = E\theta_k^{(i)} = \int_0^\infty \bar{P}_i(x)dx$.

The storage process at superposition of two renewal processes is defined in an ordinary way

$$\rho^\varepsilon(t) = u + \sum_{n=1}^{\nu(t/\varepsilon)} \alpha_n^\varepsilon\left(\kappa_n\right), \quad t \ge 0, \tag{4.5}$$

i.i.d. random variables $\alpha_n^\varepsilon(x), \quad x \in E$ are defined by distribution functions

$$\Phi_{ix}^\varepsilon(du) = P\left\{\alpha_n^\varepsilon(ix) \in du\right\}, \quad i = 1, 2$$

that satisfy Poisson approximation conditions:

PAC1: $\int_R u\Phi_{ix}^\varepsilon(du) = \varepsilon\left[a_i(x) + \theta_{ai}^\varepsilon(x)\right], \quad i = 1, 2,$

$\int_R u^2\Phi_{ix}^\varepsilon(du) = \varepsilon\left[c_i(x) + \theta_{ci}^\varepsilon(x)\right], \quad i = 1, 2,$

PAC2: $\int_R g(u)\Phi_{ix}^\varepsilon(du) = \varepsilon\left[\Phi_g(ix) + \theta_{gi}^\varepsilon(x)\right], \quad i = 1, 2,$

where $\Phi_g(ix) = \int_R g(u)\Phi_{ix}(du), \quad g(u) \in C_3\,(R)\,.$

Corollary 1. Under the conditions PAC 1-2 the weak convergence $\rho_\varepsilon(t) \Rightarrow \rho^0(t)$, $\varepsilon \to 0$ takes place.

The limit compound Poisson process $\rho^0(t), \quad t \geq 0,$ is defined by its predictable characteristics

$$B_0(t) = qb_0t, \quad C_0(t) = qc_0t, \quad \Phi_g^0(t) = q\Phi_g^0 t,$$

$$b_0 = \rho_1 \mathrm{E}a_1\,(\theta_2^*) + \rho_2 \mathrm{E}a_2\,(\theta_1^*)\,, \quad c_0 = \rho_1 Ec_1(\theta_2^*) + \rho_2 Ec_2\ (\theta_1^*),$$

$$\Phi_g^0 = \rho_1 \int_0^\infty P_2^*(x)\Phi_g(1x)dx + \rho_2 \int_0^\infty P_1^*(x)\Phi_g(2x)dx$$

Conclusions

1) Asymptotic behavior of stochastic storage processes with critical jumps in random media, described by Markov or semi-Markov processes, at increasing time intervals are approximated by compound Poisson process with continuous drift.

2) Critical stochastic events like catastrophes, large payments, etc. take place by an exponential distribution of event's time. Thus, in the models of stochastic storage processes studied here the forecast of critical events is impossible. Only statistical estimation of the intensity of critical events is possible.

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