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LÉVY PROCESSES AND ITÔ–SKOROKHOD INTEGRALS

We study Skorokhod integral processes on Lévy spaces and prove an equivalence between this class of processes and the class of Itô–Skorokhod processes (in the sense of [14]). Using this equivalence, we introduce a stochastic analysis of the Itô type for anticipating integrals on Lévy spaces.

Introduction

We study in this work the anticipating integrals with respect to a Lévy process. The anticipating integral on the Wiener space, known in general as the Skorokhod integral (and sometimes as the Hitsuda integral), constitutes an extension of the standard Itô integral to non-adapted integrands. It is nothing else than the classical Itô integral if the integrand is adapted. The Skorokhod integral has been extended to the Poisson process, and next it has been defined with respect to a normal martingale (see [3]) due to the Fock space structure generated by such processes. Recently, an anticipating calculus of the Malliavin-type has been defined on Lévy spaces again by using some multiple stochastic integrals with respect to a Lévy process which have been defined, in essence, in the old paper by K. Itô (see [4]). We refer to [8], [9], or [13] for the Malliavin calculus on Lévy spaces and possible applications to mathematical finance.

The purpose of this paper is to understand the relation between anticipating Skorokhod integral processes and Itô-Skorokhod integral processes (in the sense of [14] or [11]) in the Lévy case. We recall that the results in [14] and [11] show that, on Wiener and Poisson spaces, the class of Skorokhod integral processes with regular enough integrands coincides with the class of some Itô-Skorokhod integrals that have similar properties to the classical Itô integrals for martingales. The fact that the driven processes have independent increments plays a crucial role. Therefore, it is expected to obtain the same type of results for Lévy processes. We prove here an equivalence between Skorokhod and Itô-Skorokhod integrals by using the recently introduced Malliavin calculus for Lévy processes. Some direct consequences of the equivalence between the two classes of stochastic processes are also obtained.

Section 2 contains some preliminaries on Lévy processes and the Malliavin-Skorokhod calculus with respect to them. In Section 3, we prove a generalized Clark-Ocone formula that we will use in Section 4 to prove the correspondence between Skorokhod and Itô-Skorokhod integrals and to develop an Itô-type calculus for the anticipating integrals on Lévy spaces.

Preliminaries

In this section, we introduce the basic properties of the Malliavin calculus for Lévy processes that we will need in the paper. For more details, the reader is referred to [13].

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In this work, we deal with a càdlàg Lévy process $X = (X_t)_{0\le t\le 1}$ defined on a certain complete probability space $\left(\Omega, (F_t^X)_{0\le t\le 1}, P\right)$, with the time horizon $T = [0,1]$, and equipped with its generating triplet (γ, σ^2, ν), where $\gamma \in R$, $\sigma \ge 0$ and $\nu(dz)$ is the Lévy measure on R which, we recall, is such that $\nu(\{0\}) = 0$ and

$$\int_R 1 \wedge x^2 \nu(dx) < \infty.$$

Throughout the paper, we suppose that $\int_R x^2 \nu(dx) < \infty$, and we use the notation and terminologies as in [1], [13]. By N, we denote the jump measure of X:

$$N(E) = \#\{t : (t, \Delta X_t) \in E\},$$

for $E \in B(T \times R_0)$, where $R_0 = R - \{0\}$, $\Delta X_t = X_t - X_{t^-}$, $\#$ denotes the cardinal. We will note $\widetilde{N}$ the compensated jump measure:

$$\widetilde{N}(dt, dx) = N(dt, dx) - dt\nu(dx).$$

The process X admits a Lévy-Itô representation

$$X_t = \gamma t + \sigma W_t + \int\int_{(0,t]\times\{|x|>1\}} xN(ds, dx) + \lim_{\varepsilon\downarrow 0} \int\int_{(0,t]\times\{\varepsilon<|x|\le 1\}} x\widetilde{N}(ds, dx),$$

where W is a standard Brownian motion.

Itô [4] proved that X can be extended to a martingale-valued measure M of type $(2, \mu)$ on $(T \times R, B(T \times R))$.

For any $E \in B(T \times\ R)$ with $\mu(E) < \infty$

$$M(E) = \sigma \int_{E(0)} dW_s + \lim_{n\to\infty} \int\int_{\{(s,x)\in E:\frac{1}{n}<|x|<n\}} x\widetilde{N}(ds, dx),$$

where $E(0) = \{s \in T : (s, 0) \in E\}$ and

$$\mu(E) = \sigma^2 \int_{E(0)} ds + \int\int_{\{E-E(0)\times\{0\}\}} x^2 ds\nu(dx).$$

Furthermore, M is a centered independent random measure such that

$$E\left(M(E_1)M(E_2)\right) = \mu(E_1 \cap E_2)$$

for any $E_1, E_2 \in B(T \times R)$ with $\mu(E_1) < \infty$ and $\mu(E_2) < \infty$.

Using the random measure M, one can construct multiple stochastic integrals driven by a Lévy process as an isometry between $L^2(\Omega)$ and the space

$$L^2\left((T \times R)^n, B((T \times R)^n), \mu^{\otimes n}\right).$$

Indeed, one can use the same steps as on the Wiener space: first, consider a simple function f of the form

$$f = 1_{E_1\times\ldots\times E_n},$$

where $E_1, \ldots, E_n \in B(T \times R)$ are pathwise disjoint and $\mu(E_i) < \infty$ for every i. For a such function, we define

$$I_n(f) = M(E_1)\ldots M(E_n)),$$

and then the operator I_n can be extended by linearity and continuity to an isometry between $L^2(\Omega)$ and the space $L^2\left((T \times R)^n, B((T \times R)^n), \mu^{\otimes n}\right)$.

An interesting fact is that, as in the Brownian and Poissonian cases, M enjoys the chaotic representation property (see [13]), i.e., every $F \in L^2(\Omega, F^X, P) = L^2(\Omega)$ can be written as an orthogonal sum of multiple stochastic integrals

$$F = E(F) + \sum_{n=1}^{\infty} I_n(f_n)$$

where it converges in $L^2(\Omega)$ and $f_n \in L_s^2\left((T \times R)^n, B((T \times R)^n), \mu^{\otimes n}\right)$ (the last space is the space of symmetric and square integrable functions on $(T \times R)^n$ with respect to $\mu^{\otimes n}$.)

At this point, we can introduce the Malliavin calculus with respect to the Lévy process X by using this Fock space-type structure. If

$$\sum_{n=0}^{\infty} nn! \|f_n\|_n^2 < \infty$$

(here $\|f_n\|_n$ denotes the norm in the space $L^2\left((T \times R)^n, B((T \times R)^n), \mu^{\otimes n}\right)$), then the Malliavin derivative of F is introduced as an annihilation operator (see, e.g., [7])

$$D_z F = \sum_{n=1}^{\infty} n I_{n-1}(f_n(z, .)), \quad z \in T \times R.$$

The domain of the derivative operator D is denoted by $D^{1,2}$. It contains a random variable of the above chaotic form such that $\sum_{n=0}^{\infty} nn! \|f_n\|_n^2 < \infty$ holds. We denote, by $D^{k,2}$, $k \geq 1$, the domain of the kth iterated derivative $D^{(k)}$, which is a Hilbert space with respect the scalar product

$$\langle F, G \rangle = E(FG) + \sum_{j=1}^{k} E \int_{(T \times R)^j} D_z^{(j)} F D_z^{(j)} G \mu^{\otimes j}(dz).$$

We introduce now the Skorokhod integral with respect to X as a creation operator. Let $u \in H = L^2\left(T \times R \times \Omega, B(T \times R) \otimes F_T^X, \mu \otimes P\right)$. Then, for every $z \in T \times R$, $u(z)$ admit the representation

$$u(z) = \sum_{n=0}^{\infty} I_n(f_n(z, .)).$$

Here, we have $f_n \in L^2((T \times R)^{n+1}, \mu^{\otimes^{n+1}})$, and f_n is symmetric in the last n variables. If

$$\sum_{n=0}^{\infty} (n+1)! \|\tilde{f}_n\|_{n+1}^2 < \infty$$

($\tilde{f}_n$ represents the symmetrization of f_n in all its $n+1$ variables), then the Skorokhod integral $\delta(u)$ of u with respect to X is introduced by

$$\delta(u) = \sum_{n=0}^{\infty} I_{n+1}(\tilde{f}_n).$$

The domain of δ is the set of processes satisfying $\sum_{n=0}^{\infty} (n+1)! \|\tilde{f}_n\|_{n+1}^2 < \infty$, and we have the duality relationship

$$E(F\delta(u)) = E \int \int_{T \times R} D_z F u(z) \mu(dz), \quad F \in D^{1,2}.$$

We will use the notation

$$\delta(u) = \int_0^1 \int_R u_z \delta M(dz) = \int_0^1 \int_R u_{s,x} \delta M(ds, dx).$$

Remark 1. It has been proved in [13] that if the integrand is predictable, then the Skorokhod integral coincides with the standard semi-martingale integral introduced in [1].

For $k \geq 1$, we denote, by $L^{k,2}$, the set $L^2((T \times R; D^{k,2}), \mu)$. In particular, one can prove that $L^{1,2}$ is given by the set of u in the above chaotic form such that

$$\sum_{n=0}^{\infty} (n+1)! \|\tilde{f}_n\|_{n+1}^2 < \infty.$$

We also have $L^{k,2} \subset Dom\delta$ for $k \geq 1$ and, for every $u, v \in L^{1,2}$,

$$E(\delta(u)\delta(v)) = E \int \int_{T \times R} u(z)v(z)\mu(dz) + E \int \int_{(T \times R)^2} D_z u(z') D_{z'} v(z) \mu(dz)\mu(dz').$$

In particular,

$$E(\delta(u))^2 = E \int \int_{T \times R} u(z)^2 \mu(dz) + E \int \int_{(T \times R)^2} D_z u(z') D_{z'} u(z) \mu(dz)\mu(dz').$$

The commutativity relationship between the derivative operator and the Skorokhod integral is given by: let $u \in L^{1,2}$ such that $D_z u \in Dom(\delta)$, then $\delta(u) \in D^{1,2}$ and

$$D_z \delta(u) = u(z) + \delta(D_z(u)), \quad z \in T \times R.$$

Generalized Clark-Ocone formula

We start this section by proving some properties of the multiple integrals $I_n(f)$ and show how it behaves if it is conditioned by a σ-algebra. If $A \in B(T)$, we will denote, by F_A^X, the σ -algebra generated by increments of the process X on the set A

$$F_A^X = \sigma(X_t - X_s : s, t \in A).$$

Proposition 1. *Let $f_n \in L_s^2((T \times R)^n, \mu^{\otimes n})$ and $A \in B(T)$. Then*

$$E\left(I_n(f)/F_A^X\right) = I_n(f 1_{(A \times R)}^{\otimes n}).$$

Proof. By the density and linearity arguments, it is enough to consider $f = 1_{E_1 \times \ldots \times E_n}$, where $E_1, ..., E_n$ are a pairwise disjoint set of $B\left(T \times R\right)$ and $\mu(E_i) < \infty$ for every $i = 1, ..., n$. In this case, we have

$$E\left(I_n(f)/F_A^X\right) = E\left(M(E_1)....M(E_n)/F_A^X\right),$$

$$E\left(\prod_{i=1}^n \left(M(E_i \cap (A \times R)) + M(E_i \cap (A^c \times R))\right)/F_A^X\right)$$

$$= \prod_{i=1}^n M(E_i \cap (A \times R)) = I_n(f 1_{(A \times R)}^{\otimes^n}).$$

As an immediate consequence, we have

Corollary 1. *Suppose that $F \in D^{1,2}$ and $A \in B(T)$. Then the conditional expectation $E(F/F_A^X)$ belongs to $D^{1,2}$, and for every $z \in T \times R$, we have*

$$D_z E\left(F/F_A^X\right) = E\left(D_z F/F_A^X\right) 1_{A\times R}(z).$$

Proof. Let $F = \sum_{n\geq 0} I_n(f_n)$ with $f_n \in L_s^2\left((T\times R)^n, B((T\times R)^n), \mu^{\otimes n}\right)$. Then, by Proposition 1,

$$D_z E\left(F/F_A^X\right) = D_z\left(\sum_{n\geq 0} I_n\left(f_n 1_{A\times R}^{\otimes n}\right)\right)$$
$$= \sum_{n\geq 1} n I_{n-1}\left(f_n(\cdot, z) 1_{A\times R}^{\otimes(n-1)}\right) 1_{A\times R}(z)$$

, and it remains to observe that $E\left(D_z F/F_A^X\right) = \sum_{n\geq 1} n I_{n-1}\left(f_n(\cdot,z)1_{A\times R}^{\otimes(n-1)}\right)$.

At this point, we will state the following version of the Clark-Ocone formula on the Lévy space, which extends the results in [13].

Proposition 2. *[Generalized Clark-Ocone-Haussman formula] Let F be a random variable in $D^{1,2}$. Then, for every $0 \leq s < t \leq 1$, we have*

$$F = E\left(F/F_{(s,t]^c}^X\right) + \delta(h_{s,t}(\cdot))$$

, where, for $(r,x) \in T\times R$, we denoted $h_{s,t}(r,x) = E\left(D_{r,x}F/F_{(r,t]^c}\right) 1_{(s,t]^c}(r)$. Moreover,

$$F = E\left(F/F_{(s,t]^c}^X\right) + \int\int_{(s,t]\times R} {}^{(p,t)}(D_z F)\, dM_z$$
$$= E\left(F/F_{(s,t]^c}^X\right) + \sigma\int_s^t {}^{(p,t)}(D_{r,0}F) dW_r + \int\int_{(s,t]\times R_0} {}^{(p,t)}(D_{r,x}F)\ \widetilde{N}(dr,dx)$$

, where ${}^{(p,t)}(DF)$ is the predictable projection of DF with respect to the filtration

$$\left(F_{(r,t]^c}^X\right)_{r\leq t}.$$

Proof. Let $F = \sum_{n=0}^{\infty} I_n(f_n)$, where $f_n \in L_s^2\left(([0,1]\times R)^n, \mu^{\otimes^n}\right)$. Firstly, we prove that

$$F = E\left(F/F_{(s,t]^c}^X\right) + \delta\left(h_{s,t}(\cdot)\right).$$

Indeed, for any $s < t \leq 1$, we have

$$E(D_{r,x}F/F_{(r,t]^c}^X)1_{(s,t]\times R}(r,x) = \sum_{n=1}^{\infty} n I_{n-1}\left[f_n((r,x),.)1_{(r,t]^c\times R}^{\otimes^{n-1}}(.)1_{(s,t]\times R}(r,x)\right].$$

Hence, using that $x \in R$ and thus the symmetrization with respect to the variable x has no effect, we obtain

$$\delta(h) = \sum_{n=1}^{\infty} n I_n\left[f_n((t_1,x_1),...,(t_n,x_n))1_{(t_1,t]^c}^{\otimes^{n-1}}\widetilde{(t_2,...,t_n)}1_{(s,t]}(t_1)\right]$$

Since

$$1_{(t_1,t]^c}^{\otimes^{n-1}}\widetilde{(t_2,...,t_n)}1_{(s,t]}(t_1)$$
$$= \frac{1}{n!}\sum_{i=1}^{n}\sum_{\sigma(1)=i,\sigma\in S_n}^{n} 1_{(t_i,t]^c}^{\otimes^{n-1}}(t_{\sigma(2)},...,t_{\sigma(n)})1_{(s,t]}(t_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} 1_{(t_i,t]^c}^{\otimes^{n-1}}(t_1,...,\widehat{t_i},...,t_n)1_{(s,t]}(t_i)$$

$$= \frac{1}{n}\left(1 - 1_{(s,t]^c}^{\otimes^{n}}(t_1,...,t_n)\right),$$

we have

$$\delta\left(h_{s,t}\right) = F - E\left(F/F^X_{(s,t]^c}\right).$$

The second equality in the statement follows from [13] (see also [11]), where the equivalence of the two representations has been proven.

Itô-Skorokhod integral calculus

As a consequence of the above results, we will show in this part that every Skorokhod integral process of the form

$$Y_t := \delta(u_. 1_{[0,t]\times R}(\cdot)), \qquad t \in T,$$

can be written as an Itô-Skorokhod integral in the sense of [14] (this integral has similarities with the standard stochastic integral). In this way, we will extend the results of [14] to the Wiener case and those of [11] to the Poisson case. The key point of our construction is the fact that the driving process has independent increments.

Proposition 3. *Assume that $u \in L^{k,2}$ with $k \geq 3$. Then there exists a unique process $v \in L^{k-2,2}$ such that, for every $t \in T$,*

$$Y_t := \delta(u_. 1_{[0,t]\times R}(\cdot)) = \int\int_{(0,t]\times R}{}^{(p,t)}(v_{s,x})\ M(ds,dx).$$

Moreover, $v_{s,x} = D_{s,x}Y_s \quad \mu\otimes P$ a.e. on $T\times R\times\Omega$.

Proof. Applying above generalized Clark-Ocone formula, we have

$$Y_t = E\left(Y_t/F^X_{t^c}\right) + \int\int_{(0,t]\times R}{}^{(p,t)}(D_z Y_t)\ dM_z.$$

The process Y satisfies

$$E\left(Y_t - Y_s/F^X_{(s,t]^c}\right) = 0$$

for every $s < t$. Indeed, we take a random variable F $F^X_{(s,t]^c}$-measurable in $D^{1,2}$. According to the duality relationship and Corollary 1, we have

$$E(F(Y_t - Y_s)) = E[\delta(u_. 1_{(s,t]\times R}(.))] = E\left\langle D_. F, u_. 1_{(s,t]\times R}(.)\right\rangle_{L^2(T\times R,\ \mu)} = 0.$$

Therefore, we obtain

$$E\left(Y_t/F^X_{t^c}\right) = E\left(Y_t - Y_0/F^X_{t^c}\right) = 0$$

and

$$\delta\left[E^{\ (p,t)}\left(D_{s,x}Y_t\right)1_{[0,t]\times R}(s,x)\right] = \delta\left[E\left(D_{s,x}Y_t/F^X_{(s,t]^c}\right)1_{[0,t]\times R}(s,x)\right]$$

$$= \delta\left[D_{s,x}E\left(Y_t/F^X_{(s,t]^c}\right)1_{[0,t]\times R}(s,x)\right]$$

$$= \delta\left[D_{s,x}E\left(Y_s/F^X_{(s,t]^c}\right)1_{[0,t]\times R}(s,x)\right]$$

$$= \delta\left[E\left(D_{s,x}Y_s/F^X_{(s,t]^c}\right)1_{[0,t]\times R}(s,x)\right]$$

$$= \delta\left[E^{\ (p,t)}\left(D_{s,x}Y_s\right)1_{[0,t]\times R}(s,x)\right]$$

We thus have

$$Y_t = \int\int_{(0,t]\times R}^{(p,t)} (D_{s,x}Y_s)\ M(ds,dx).$$

Set $v_{s,x} = D_{s,x}X_s$. To obtain the desired Itô-Skorokhod representation, it is sufficient to prove that $v \in L^{k-2,2}$. By using the property of commutativity between D and δ and the inequalities for the norms of anticipating integrals, we have

$$\|v\|_{1,2}^2 \le \|u\|_{1,2}^2 + \|\delta(D_{s,x}u_{\cdot}1_{[0,s]\times R}(\cdot))\|_{1,2}^2$$

$$\le \|u\|_{1,2}^2 + E\int\int_{T\times R} \left(\delta(D_{s,x}u_{\cdot}1_{[0,s]\times R}(\cdot)\right)^2 \mu(ds,dx)$$

$$+E\int\int_{(T\times R)^2} \left(D_{r,y}\delta(D_{s,x}u_{\cdot}1_{[0,s]\times R}(\cdot))\right)^2 \mu(ds,dx)\mu(dr,dy)$$

$$\le \|u\|_{1,2}^2 + 3E\int_{T\times R}\int_{T\times R} (D_{z_2}u_{z_1})^2 \mu(z_1)\mu(z_2)$$

$$+3E\int_{T\times R}\int_{T\times R}\int_{T\times R} (D_{z_3}D_{z_2}u_{z_1})^2 \mu(z_1)\mu(z_2)\mu(z_3)$$

$$+2E\int_{T\times R}\int_{T\times R}\int_{T\times R}\int_{T\times R} (D_{z_4}D_{z_3}D_{z_2}u_{z_1})^2 \mu(z_1)\mu(z_2)\mu(z_3)\mu(z_4) \le 4\|u\|_{3,2}^2.$$

In the same manner, we found that

$$\|v\|_{k-2,2}^2 \le C(k)\|u\|_{k,2}^2,$$

where $C(k)$ is a positive constant depending of k. To conclude our proof, we have to show the uniqueness of these processes. We assume that there exist v and v' in $L^{k-2,2}$ such that

$$Y_t = \int\int_{(0,t]\times R}^{(p,t)} (v_{s,x})\ M(ds,dx) = \int\int_{(0,t]\times R}^{(p,t)} (v'_{s,x})\ M(ds,dx).$$

Using again the property of commutativity, we have

$$E\left(w_{s,x}/F^X_{[s,t]^c}\right)1_{[0,t]\times R}(s,x) + \int\int_{(s,t]\times R} E\left(D_{s,x}w_{r,x}/F^X_{[r,t]^c}\right)\ M(dr,dx) = 0,$$

where $w_{s,x} = v_{s,x} - v'_{s,x}$. Conditioning by $F^X_{[s,t]^c}$, we obtain

$$E\left(w_{s,x}/F^X_{[s,t]^c}\right)1_{[0,t]\times R}(s,x) = 0, \quad s \le t, x \in R.$$

By letting t goes to s, we get that $w_{s,x} = 0$ in $L^2(\Omega)$ for every $(s,x) \in\ T\times\ R$. We can thus conclude that $v = v'$ in $L^{k-2,2}$.

Using the correspondence between Skorokhod and Itô-Skorokhod integrals proved above, we can derive the Itô formula for anticipating integrals on the Lévy space. As far as we know, this is the only Itô formula proved for this class of stochastic processes.

Proposition 4. *[Itô's formula] Let v be a process belonging to $L^2(T \times R \times \Omega, \mu \otimes P)$. Let us consider the stochastic process*

$$Y_t = \int\int_{(0,t]\times R} E\left(v_{s,x}/F^X_{[s,t]^c}\right) M(ds,dx),$$

and let f be a C^2 real function. Then

$$f(Y_t) = f(0) + \int\int_{(0,t]\times R} f'(Y_t^{s^-})^{(p,t)}(D_{s,x}Y_s)\ M(ds,dx)$$

$$+\frac{1}{2}\int\int_{(0,t]\times R} f''(Y_t^{s^-})(^{(p,t)}(D_{s,0}Y_s))^2\ ds$$

$$+\sum_{0<s\leq t}(f(Y_t^s) - f(Y_t^{s^-}) - f'(Y_t^{s^-})(Y_t^s - Y_t^{s^-})),$$

where $Y_t^s := \int\int_{(0,s]\times R} E\left(v_{s,x}/F^X_{[s,t]^c}\right) M(ds,dx)$ and $Y_t^{s^-} = \lim_{r\to s^-} Y_t^r$ for all $0 < s \leq t$.

Proof. Fix $t \in (0,T]$. We define $Z_s = Y_t^s$ if $s \leq t$ and $Z_s = Y_t$ if $s > t$. Also let $(G_s)_{s\geq 0}$ be a filtration given as follows: $G_s = F^X_{[s,t]^c}$ if $s \leq t$ and $G_s = F^X_1$, if $s > t$.

It is easy to see that $(Z_s)_{s\geq 0}$ is a square integrable càdlàg martingale with respect to $(G_s)_{s\geq 0}$; therefore, the standard stochastic calculus for jump processes can be applied to it.

Applying Itô's formula (see [12], Theorem 32, p. 71), we obtain, for every $s > 0$,

$$f(Z_s) = f(0) + \int_{(0,t]} f'(Z_{s^-})dZ_s + \frac{1}{2}\int_{(0,t]} f''(Z_{s^-})d[Z,Z]^c_s$$

$$+\sum_{0<s\leq t}(f(Z_s) - f(Z_{s^-}) - f'(Z_{s^-})(Z_s - Z_{s^-})),$$

where $[Z,Z]^c$ is the continuous part of the quadratic variation process $[Z,Z]$ of Z. It is well known that $[N,N]^c_s = 0$. From Proposition 4 of [13], we see that, for every $s \leq t$,

$$[Z,Z]^c_s = [Y,Y]^c_s = \sigma^2 \int_{(0,s]} E\left(v_{r,0}/F^X_{[r,t]^c}\right)^2 dr.$$

Thus, and in particular for $s = t$ (in the sense of the limit almost sure or in L^2)),

$$f(Z_t) = f(Y_t) = f(0) + \int\int_{(0,t]\times R} f'(Y_t^{s^-})E\left(v_{s,x}/F^X_{[s,t]^c}\right)\ M(ds,dx)$$

$$+\frac{1}{2}\int\int_{(0,t]\times R} f''(Y_t^{s^-})\left[E\left(v_{s,0}/F^X_{[s,t]^c}\right)\right]^2\ ds$$

$$+\sum_{0<s\leq t}(f(Y_t^s) - f(Y_t^{s^-}) - f'(Y_t^{s^-})(Y_t^s - Y_t^{s^-})).$$

So that the result holds.

Another consequence of Proposition 3 is the following Burkholder inequality which gives a bound for the L^p norm of the anticipating integral.

Proposition 5. *[Burkholder's Inequality] Let Y be a process of the Itô-Skorokhod form as above, and let $2 \leq p < \infty$. Then there exist a universal constant $C(p)$ such that*

$$E|Y_t|^p \leq C(p)E\Bigg(\sigma^2 \int_{(0,t]} E\left(v_{r,0}/F^X_{[r,t]^c}\right)^2 dr + \int\int_{(0,t]\times R_0} x^2 E\left(v_{r,x}/F^X_{[r,t]^c}\right)^2 \; N(dr,dx)\Bigg)^{p/2}.$$

Proof. The proof of this proposition is straightforward from Theorem 54 in [12, p. 174] and the approximation procedure used along the paper.

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